Exam 2 Calc 2 2/28/2014

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Derive the formula $\int xe^x dx = xe^x - e^x + C$.

$\int xe^x dx$

$u = x \quad v = e^x$

$u' = 1 \quad v' = e^x$

$uv - \int u'v$ is the rule, so,

$x \cdot e^x - \int 1 \cdot e^x \to xe^x - e^x + C$

Great!

2. Give the form for a partial fractions decomposition of $\int \frac{2(x^4+1)}{(x-2)(x^2-1)^2(x^2+2)^2} dx$,

or explain why one does not exist.

$$\frac{A}{x-2} + \frac{B}{(x-1)} + \frac{C}{(x+1)} + \frac{D}{(x-1)^2} + \frac{E}{(x+1)^2} + \frac{Fx+G}{x^2+2} + \frac{Hx+I}{(x^2+2)^2}$$

3. Suppose you're approximating $\int_3^7 f(x)\,dx$, and that the left-hand approximation with 10 subdivisions is 1.7034, the right-hand approximation with 10 subdivisions is 1.9282, and the midpoint approximation with 10 subdivisions is 1.8566. What are the trapezoidal and Simpsons approximations with 10 subdivisions for this integral?

L_n	R_n	M_n	T_n	S_n
1.7034	1.9282	1.8566	$(L_n+R_n)/2$	$(2M_n+T_n)/3$
			1.8158	1.843

Good

4. Evaluate $\int \tan^4\theta\, d\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$$\int \tan^4\theta\, d\theta = \int \tan^2\theta(\sec^2\theta - 1)\,d\theta$$

$$= \int \tan^2\theta \cdot \sec^2\theta\, d\theta - \int \tan^2\theta\, d\theta$$

$$= \int \tan^2\theta \sec^2\theta\, d\theta - \int (\sec^2\theta - 1)\,d\theta$$

$$= \int u^2 \cdot du \quad - \tan\theta + \theta + C$$

$$= \frac{1}{3}\tan^3\theta - \tan\theta + \theta + C$$

5. Evaluate $\int_8^{\infty} \frac{dx}{\sqrt[3]{x}}$.

$$\lim_{b\to\infty} \int_8^b \frac{1}{\sqrt[3]{x}}\,dx$$

$$\int_8^b x^{-\frac{1}{3}}\,dx$$

$$\left(\frac{3}{2}x^{\frac{2}{3}}\right)\Big]_8^b$$

$$\lim_{b\to\infty} \left(\frac{3}{2}b^{\frac{2}{3}}\right) - (6)$$

$$\infty - 6$$

$$\boxed{\text{diverges}}$$

Excellent

6. Evaluate $\int x \arctan(6x)\,dx$.

$u = 6x$ $\quad x = \frac{1}{6}u$

$du = 6dx$

$$\frac{1}{36}\int u \arctan u\,du$$

$v = \tan^{-1} u$ $\quad w = \frac{1}{2}u^2$

$v' = \frac{1}{u^2+1}$ $\quad w' = u$

$$\frac{1}{36}\left[\frac{1}{2}u^2 \tan^{-1} u - \int \frac{\frac{1}{2}u^2}{u^2+1}\,du\right]$$

$$\underbrace{\frac{1}{72}(6x)^2 \arctan(6x)}_{\frac{36x^2}{72} = \frac{1}{2}x^2} - \frac{1}{72}\int \frac{u^2}{u^2+1}\,du$$

Wonderful!

$$-\frac{1}{72}\int \frac{u^2+1}{u^2+1} - \frac{1}{u^2+1}\,du$$

$$-\frac{1}{72}u + \frac{1}{72}\int \frac{1}{u^2+1}\,du$$

$$\underbrace{-\frac{1}{72}(6x)}_{-\frac{1}{12}x} \qquad \frac{1}{72}\tan^{-1} u$$

$$\underbrace{\frac{1}{72}\arctan(6x)}$$

$$\boxed{\frac{1}{2}x^2 \arctan(6x) + \frac{1}{72}\arctan(6x) - \frac{1}{12}x + C}$$

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod! We were doing our homework last night, and there was this one problem, like integrating tangent of *e* to the *x*, right? And the table of integrals has tan *x*, but can you just use that with *e* to the *x* where they have the *x*? It seems too easy."

   Help Bunny by explaining whether such an approach works. You do *not* need to actually work the integral out, but explain to Bunny whether her plan is valid.

$$\int \tan(e^x)\,dx$$

No Bunny, you can't. "X" and "e^x" do not have the same derivatives. If you used that line from the table you would get the wrong answer because "x" has a derivative of 1. "e^x" has a derivative of e^x so you would need to divide the entire integral by e^x, leaving you with a not very pretty problem. So please do not rely on the table of integrals to always hand you the right answer! You need to understand when it is appropriate to use the table and when it is not.

Excellent

8. Evaluate $\int \frac{x^4}{1+x^2}\,dx$. $= \int \frac{\tan^4\theta}{1+\tan^2\theta} \times \sec^2\theta \, d\theta$

$x = \tan\theta$

$\frac{dx}{d\theta} = \sec^2\theta$

$dx = \sec^2\theta \, d\theta$

$\int \frac{\tan^4\theta}{\cancel{\sec^2\theta}} \times \cancel{\sec^2\theta}\, d\theta$

$= \int \tan^4\theta \, d\theta$ [now, using line 43 from table of integrals]

$= \frac{\tan^3\theta}{3} - \tan\theta + \theta + C$

~~reverse~~ sub for $x = \tan\theta$ & $\theta = \tan^{-1}x$

$\frac{x^3}{3} - x + \tan^{-1}x + C$

Nice Job.

9. Evaluate $\int \frac{1}{x^3 - 6x^2}\,dx$.

Partial Fractions!

I wish:

$$\frac{1}{x^2(x-6)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-6}$$

$$1 = A(x)(x-6) + B(x-6) + Cx^2$$

$$1 = Ax^2 - 6Ax + Bx - 6B + Cx^2$$

Matching constants:

$$1 = -6B$$

$$\therefore B = -1/6$$

Matching x coefficients:

$$0 = -6A + B$$

$$0 = -6A + -1/6$$

$$\frac{1}{6} = -6A$$

$$A = \frac{-1}{36}$$

Matching x^2 coefficients:

$$0 = A + C$$

$$0 = \frac{-1}{36} + C$$

$$C = \frac{1}{36}$$

So $$\int \frac{1}{x^3 - 6x^2}\,dx = \int \left(\frac{-1/36}{x} + \frac{-1/6}{x^2} + \frac{1/36}{x-6} \right) dx$$

$$= \frac{-1}{36} \cdot \ln|x| - \frac{1}{6} \cdot -x^{-1} + \frac{1}{36} \cdot \ln|x-6| + C$$

$$= \frac{-1}{36} \ln|x| + \frac{1}{6x} + \frac{1}{36} \ln|x-6| + C$$

$\tan^2 x + 1 = \sec^2 x$

10. Derive the formula $\int \tan^n x\,dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx$ (for $n \neq 1$), line 43 from the table of integrals. $\sec^2 x$ is even

$\int (\tan^{n-2} x)(\sec^2 x - 1)\,dx$

$\int \sec^2 x \tan^{n-2} x\,dx - \int \tan^{n-2} x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$\int u^{n-2}\,du - \int \tan^{n-2} x\,dx$

$\left(\frac{1}{n-1}\right) u^{n-1} - \int \tan^{n-2} x\,dx$

$\boxed{\frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x\,dx}$

Wonderful!