Exam 3 Calc 2 4/4/2014

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Write the 3rd degree Maclaurin polynomial for e^x.

2. Determine whether $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges.

3. Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converges or diverges.

4. Determine whether the series $\sum_{k=1}^{\infty} \frac{k^2 - 1}{k^3 + 4}$ converges or diverges.

5. Determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ converges or diverges.

6. Determine the radius of convergence of the power series $\sum \left(\frac{x}{3}\right)^k$.

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Well, crap. I'm getting okay at finding these Taylor series and stuff, 'cause I found there's a formula in the book. But then there's all these other things they bring in and I'm pretty lost. I might have to kill my roommate, 'cause they say you get all A's for a semester if your roommate dies. But if I can figure stuff out by the exam tomorrow, I guess I won't have to do that. So like one of the things the prof said we needed to know was why the series x to the n sums up to 1 over 1 minus x, and he said it was more an explaining thing about reasons than a bunch of calculating, but I'm not so good with reasons. Maybe I need to think more about the roommate option..."

   Help Biff (and his roommate!) by explaining clearly how we can find the sum of $\sum x^n$.

8. Use a Taylor series with at least 4 nonzero terms to approximate $\sqrt{e}$.

9. Use a Taylor series with at least 3 nonzero terms to approximate $\int_0^{0.2} \sin\left(x^2\right) dx$.

10. Use a Taylor series to evaluate $\lim_{x \to 0} \frac{x}{e^x - e^{-x}}$

Extra Credit (5 points possible):

Find a power series representation for $\ln\left(\frac{1+x}{1-x}\right)$.