

Exam 1 Differential Equations 2/14/14

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Is $y(t) = t^2 - 2t + 2 + 5e^{-t}$ a solution to the differential equation $\frac{dy}{dt} = -y + t^2$?

2. Find a general solution to the differential equation $\frac{dy}{dt} = ty$.

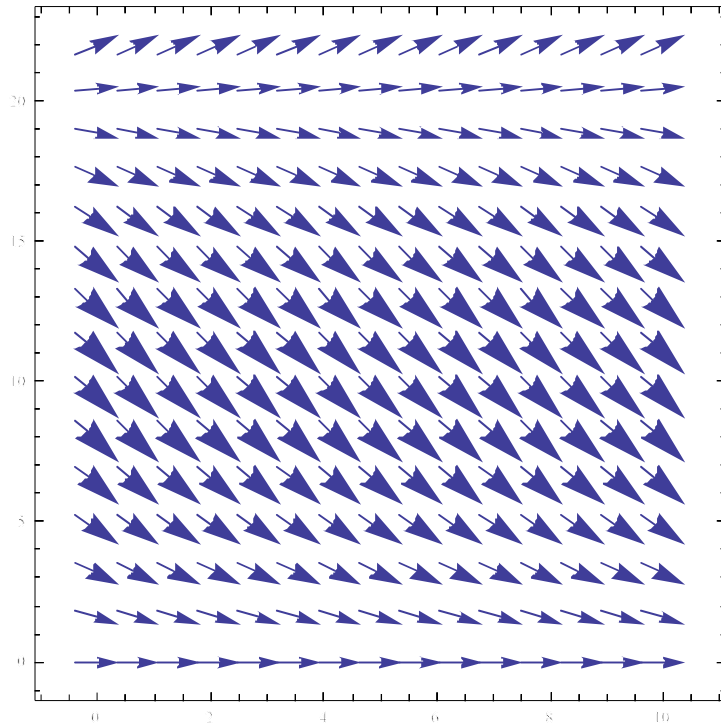
3. Sketch a phase line for the differential equation $\frac{dP}{dt} = 0.0037P(4000 - P)$.

4. Consider the differential equation $\frac{dy}{dt} = y - 4t + y^2 - 8yt + 16t^2 + 4$. Change the dependent variable from y to u using the change of variables $u = y - 4t$.

5. Consider the differential equation $\frac{dT}{dt} = 0.02(30 - T)$, with initial condition $T(0) = 170$, modeling the temperature of a cup of coffee sitting in a car's cupholder. Use Euler's method with $\Delta t = 5$ to approximate the temperature of the coffee after 10 minutes, when the driver spills it on his pants just as he arrives at his Valentine's date.

6. Bunny is taking Differential Equations at Enormous State University, and is having some trouble. Bunny says “Ohmygod! We’re supposed to use computers for this class, and I’m so totally lost! I turned this slope thingy in for this one where we were supposed to make it, like, logistic population, right? So where there’s carry capacity and stuff? And the grader told me no credit because it’s obviously wrong, but I typed it exactly like it was in the book. How can he tell it’s wrong anyway?”

Explain clearly to Bunny which characteristics of this slope field fit a logistic growth model, and which do not.



7. Find the power series expansion for the general solution up to degree four to the differential equation $\frac{dy}{dt} = -2ty$.

8. Find a general solution to the differential equation $\frac{dx}{dt} - 2x = 3k_2 e^{-4t}$.

9. **Sketch the bifurcation diagram** for the differential equation $\frac{dy}{dt} = y^2 - 6y + \mu$. Include direction arrows on the phase lines and make clear the exact μ values where bifurcations occur.

10. The bunnies on Valentine's Island have an unusual genetic variation that gives them all pink fur. When tourism begins on the island, the bunny population is 30,000. The bunny population grows 6% each year through natural reproduction. Tourists smuggle bunnies off the island, first at a modest rate of 100 bunnies per year in year 1, but then growing by 100 bunnies each year, so that by year 10, there are 1000 bunnies smuggled off the island, and so forth.
- Write a differential equation for the rabbit population on Valentine's Island.
 - Find a general solution to this differential equation.
 - Find the particular solution satisfying this initial condition.