

Exam 2 Differential Equations 3/21/14

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Does the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= 3x + y \\ \frac{dy}{dt} &= -2x\end{aligned}$$

have $x(t) = e^{2t}$, $y(t) = -2e^{2t}$ as a solution?

2. State the definition of the Laplace transform for a function $y(t)$ with at most exponential growth.

3. Construct a system of differential equations, with all coefficients representing positive constants, to model the interaction of two populations where:
- ▶ The first population would experience logistic growth with carrying capacity K in the absence of the second
 - ▶ Interaction between the two populations hurts the first population
 - ▶ The second population would experience exponential decline in the absence of the first
 - ▶ Interaction between the two populations benefits the second population
 - ▶ A fixed number of the second population are harvested in each unit of time

4. Consider the system
$$\frac{dR}{dt} = 2\left(1 - \frac{R}{3}\right)R - RF$$
 . Find all equilibrium points of this system.
- $$\frac{dF}{dt} = -16F + 4RF$$

5. Consider the system $\frac{dx}{dt} = x + 2y$. Use Euler's method with a step size of $\Delta t = 0.5$ to project $x(1)$ if $x(0) = 2$ and $y(0) = 3$.
- $\frac{dy}{dt} = -y$

6. What is the Laplace transform of $y(t) = 0$?

7. Let $y(t) = 5t$. Compute the Laplace transform of $y(t)$ from the definition.

8. Consider the system $\frac{dR}{dt} = -\frac{1}{2}F + 5R$. Find a non-trivial solution to this system.

$\frac{dF}{dt} = 8R$

9. Let $y(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } 0 \leq t < 10 \\ 2 & \text{if } 10 \leq t \end{cases}$. What is the Laplace transform of y ?

10. Suppose that $(x(t), y(t))$ is a solution to the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= \alpha x + \beta y \\ \frac{dy}{dt} &= \gamma x + \delta y\end{aligned}$$

Is it possible to say whether $(3x(t), 3y(t))$ is a solution or not? Be clear about your reasoning.