Problem Set 8 **Foundations** **Due 4/7/14**

Four of these problems will be graded, with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. (From Devlin, *Sets, Functions, and Logic*) The Hilbert Hotel, named after the famous German mathematician (and set theory pioneer) David Hilbert, has an inifinte number of rooms. The rooms are numbered 1, 2, 3, etc. One eveing — it was a Super Bowl weekend — all the rooms were booked. But late that night, an unexpected VIP turned up and asked for a room. Fortunately, the desk clerk had been reading Keith Devlin's book *Sets, Functions, and Logic*. With a bit of rearranging, the clerk was able to arrange matters so that the VIP got a room and so did everyone else. How did he do it?

2. (From Devlin, *Sets, Functions, and Logic*) The following evening the rooms were all still booked, but an infinite number of new guests arrived, all without reservations. The clever desk clerk was again able to arrange matters so that all the new arrivals got rooms and so too did the guests who had advance reservations. What did the clerk do this second night?

3. Consider the relation $\sim$ on $\mathbb{Z}$ given by $x \sim y$ iff 6 divides $x - y$. Give 3 elements of $\mathbb{Z}$ that are related to 5 and 3 elements of $\mathbb{Z}$ that are not related to 5.

4. Determine whether the relation from #3 is reflexive, symmetric, and transitive.

5. Consider the relation $\asymp$ on the set $R(I)$ of integrable functions from [0,1] to $\mathbb{R}$ defined by $f \asymp g$ iff $\int_0^1 f(x)dx = \int_0^1 g(x)dx$. Give 3 elements of $R(I)$ that are related to 5 and 3 elements of $R(I)$ that are not related to 5.

6. Determine whether the relation from #5 is reflexive, symmetric, and transitive.

7. Let $S = \{1, 2, 3, 4\}$. Then $R = \{(1, 1), (1, 3), (3, 1), (2, 2), (2, 4), (3, 3), (4, 2), (4, 4)\}$ is a relation on S. Write the equivalence classes of S associated with R.

8. Let $S = \{1, 2, 3, 4\}$. Suppose that $\sim$ is an equivalence relation with $1 \sim 2$ and $2 \sim 4$. What are the possible partitions associated with $\sim$?

9. Do the BinaryGateway on WeBWorK, available via

   http:webwork.coe.edu/webwork2/MTH-215/BinaryGateway/ .