Examlet 2 Advanced Geometry 3/4/15

1. a) State the definition of an *exterior angle* of a triangle.

Given a triangle, an exterior angle is one which forms a linear pair with one of the interior angles.

b) State the definition of a convex quadrilateral.

Given a quadrilateral, it's convex iff each vertex is in the interior of the angle formed by the other three points.

c) State the Saccheri-Legendre Theorem

For any triangle $\triangle ABC$, the angle sum $\sigma(\triangle ABC) \leq 180°$

d) State Euclid's Postulate V.

If ℓ + ℓ' are two lines cut by a traversal t in such a way that the sum of the measures of the two interior angles on one side of t is less than 180°, then ℓ + ℓ' intersect on that side of t.

Great

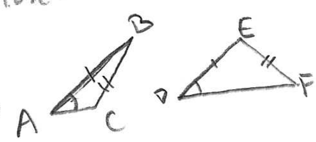
e) State the Universal Hyperbolic Theorem

If $\exists$ line ℓ_0, external point P_0 + at least two lines through P_0 + are parrel to ℓ_0, then $\forall$ lines ℓ + external point P $\exists$ at least two line through P parallel to ℓ.

Great

2. How do you know ASS (Angle-Side-Side) is not a valid triangle congruence condition?

If a triangle is not a right triangle, then ASS does not hold because two different triangles can be drawn.



△ABC and △DEF have one angle and two sides congruent as ASS describes, but the two triangles are not congruent because the angle between the two congruent sides can be acute (like △ABC), or obtuse (like △DEF). This only works in a case like the hypotenuse leg theorem with a right triangle.

Excellent!

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3. a) Provide good justifications in the blanks below for the corresponding statements:

Proposition: Let ΔABC be a triangle. If $AB > BC$ then $\mu(\angle ACB) > \mu(\angle BAC)$.

Statement:	Reason:
Let A, B, and C be three noncollinear points. Let $AB > BC$.	hypothesis,
Since $AB > BC$, there exists a point D between A and B such that $\overline{BD} \cong \overline{BC}$.	Ruler Postulate.
Now $\mu(\angle ACB) > \mu(\angle DCB)$, 1 2	Protractor Post.
and $\mu(\angle DCB) \cong \mu(\angle CDB)$. 2 3	Isoceles triangle theorem
But $\angle CDB$ is an exterior angle for ΔADC, so $\mu(\angle CDB) >$ $(\angle CAB)$.	Exterior angle theorem
The conclusion follows from those inequalities.	The conclusion follows from those inequalities.

Good

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b) Let ΔABC be a triangle. Show that if $\mu(\angle ACB) > \mu(\angle BAC)$ then $AB > BC$.
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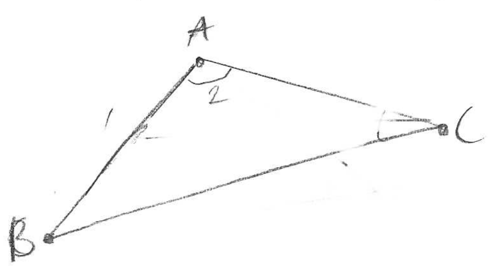

Let ΔABC be 3 noncollinear points. Let $M(\angle ACB) > M(\angle BAC)$. By the above proof we cannot have $AB < BC$ or else $M(\angle ACB) < M(\angle BAC)$ which is a contradiction. If $AB \cong BC$ then the triangle is isoscelles & $M(\angle ACB) = M(\angle BAC)$ which is also a contradiction so $AB > BC$

Nice.

4. Provide good justifications in the blanks below for the corresponding statements:

Universal Hyperbolic Theorem
~~Scalene Inequality:~~ If there exists one line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0, then for every line ℓ and for every external point P there exist at least two lines that pass through P and are parallel to ℓ.

Statement:	Reason:
S'pose there exists a line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0.	Hypothesis
Then the Euclidean Parallel Postulate fails.	Euclidean Parallel Postulate says the hypothesis does not happen
No rectangle exists.	Clairaut's Axiom is equivalent to the Euclidean Parallel Postulate
Let ℓ be a line and P an external point.	Hypothesis
We must prove that there are at least two lines through P that are both parallel to ℓ. Drop a perpendicular to ℓ through P and call the foot of that perpendicular Q.	Existence and Uniqueness of Perpendiculars
Let m be the line through P that is perpendicular to $\overleftrightarrow{PQ}$.	Existence and Uniqueness of Perpendiculars
Choose a point R on ℓ that is different from Q and let t be the line through R that is perpendicular to ℓ.	Existence and Uniqueness of Perpendiculars
Drop a perpendicular from P to t and call the foot of the perpendicular S.	Existence and Uniqueness of Perpendiculars
Now $\square PQRS$ is a Lambert quadrilateral.	Definition of Lambert Quadrilateral
But $\square PQRS$ is not a rectangle (reason?), so $\angle QPS$ is not a right angle and $\overleftrightarrow{PS} \neq m$.	No rectangle exists
Nevertheless $\overleftrightarrow{PS}$ is parallel to ℓ,	Alternating Interior Angles Theorem
so our proof is complete.	Because our proof is complete.

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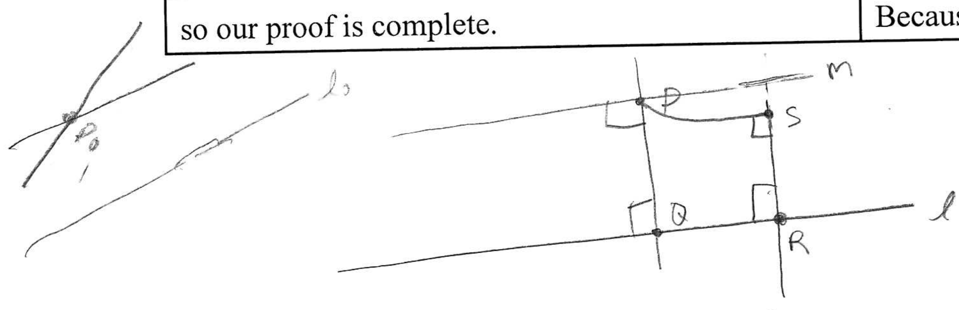
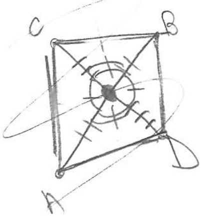


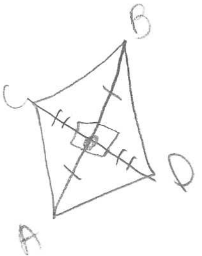
Excellent!

5. A **rhombus** is a quadrilateral with four congruent sides, and a **square** is a quadrilateral with four congruent sides and four right angles.

a) Do rhombi exist in neutral geometry? [Hint: Let $\overline{AB}$ and $\overline{CD}$ be segments that share a common midpoint, with $\overline{AB} \perp \overline{CD}$, and look at $\square ACBD$.]



They do exist. By drawing out the hint, you can see that the quadrilateral is broken into 4 triangles. The triangles are congruent to each other by SAS. Therefore the original segments of the quadrilateral: $\overline{AC}$, $\overline{CB}$, $\overline{BD}$, + $\overline{DA}$ are congruent by the def of congruent triangles. That gives you that the four sides of the quadrilateral are congruent



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∴ rhombus Great!

b) Do squares exist in neutral geometry?

Sometimes. If the Euclidean Parallel Postulate holds, then Clairaut's Axiom holds, and if rectangles exist then the Ruler Postulate, existence of perpendiculars, and non-alternating interior angles let us make squares.

But if instead the Hyperbolic Parallel Postulate holds, then no, because quadrilateral angle sums are less than 360°.