

**Exam 1    Calc 2    1/29/2016**

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Set up an integral for the area of the region bounded between  $y = 2x$  and  $y = x^2$ .

2. Evaluate  $\int \frac{x}{\sqrt{x^2 + 9}} dx$ .

3. Suppose the region between  $y = \sqrt{x}$  and the  $x$ -axis on the interval from  $x = 0$  to  $x = 9$  is rotated around the  $x$ -axis. Set up an integral for the volume of the solid produced.

4. Set up an integral for the average value of the “bump” function  $y = \frac{1}{4 + x^2}$  on the interval  $[-2, 2]$ .

5. Write an integral for the volume obtained when the region bounded between  $y = x - x^{12}$  and the  $x$ -axis is rotated about the  $y$ -axis.

6. Evaluate  $\int \frac{(\ln x)^4}{x} dx$ .

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Whoa! Calc 1 was okay, but now in Calc 2 there's all these guys who just know a lot more stuff than me and it's not even fair. On our first exam I kinda failed, and after it there were these two guys who sit in front of me joking about how easy it was 'cause it was multiple choice and they knew a bunch of the answers couldn't work so they knew the right answer without even working it out. I'm really pissed 'cause I spent like ten minutes on that one and still couldn't get it. So how the heck was I supposed to know without working it all out that the integral of  $\ln x$  from 1 to 5 is less than 7 so all the other choices are wrong?"

Help Biff out. Explain to him as clearly as possible how other students might have drawn their conclusion.

8. Take the region enclosed between  $y = 2\sin(x)$  and  $y = 4\cos(x)$  from  $x = 0$  to  $x = 0.7\pi$ , and rotate it around the line  $x = \pi$ . Write an integral for the volume of the resulting solid.

9. Jon is planning for the apocalypse by burying a spherical tank in his yard as an emergency water supply. The tank will be 1m in radius, with a tube leading up to a level 3m above the top of the tank. Jon needs to know how much work is required to pump all of the water in the tank out through the tube, so he can determine the appropriate battery to power the pump. Set up an integral for this amount of work.

10. Let  $a > 0$ . Show that the volume obtained when the region between  $y = a\sqrt{x - ax^2}$  and the  $x$ -axis is rotated around the  $x$ -axis is independent of the constant  $a$ .

Extra Credit (5 points possible): An oblate spheroid (which is roughly the shape of the planet Earth) is obtained by rotating the region inside  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  around the  $y$ -axis. Set up an integral and use it to find the volume. [Hint: as a warm-up, see what you can do with  $a$  and  $b$  just being 1.]

