

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. [Baker 5.2.6] Let X be a connected topological space with $a, b \in X$ and let $f : X \rightarrow \mathbb{R}$ be a continuous function (where \mathbb{R} has the usual topology). Prove that if y is any number between $f(a)$ and $f(b)$, then there is an element $x \in X$ such that $f(x) = y$. (This is a slightly stronger version of the Intermediate Value Theorem.)
2. [Baker 5.2.7] Complete the proof of Theorem 5.2.1.
3. [Baker 5.2.13] Determine if The Intermediate Value Theorem holds if the \mathcal{U} -relative topology on $[a, b]$ is replaced by the \mathcal{H} -relative topology.
4. [Baker 5.2.21] Let $X = \{a, b, c\}$ and $\mathcal{T} = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Find all cut points of (X, \mathcal{T}) .
5. [Baker 5.2.22] Let X and Y be topological spaces and let $f : X \rightarrow Y$ be a homeomorphism. If x is a cut point of X , then $f(x)$ is a cut point of Y .
6. [Baker 5.2.23] In the previous statement, determine if the requirement that f be a homeomorphism can be replaced by the condition that f be a continuous function.