Exam 1 **Foundations of Advanced Math** **2/3/17**

1. The sum of two throddodd integers is throdd.

To see whether this is true, we start with two throddodd integers, n and m.

By definition of throddodd, $n = 3x+2$ and $m = 3y+2$ where $x, y \in \mathbb{Z}$.

$(3x+2)+(3y+2) = 3x+3y+3+1 = 3(x+y+1)+1$

$(x+y+1)$ is an integer by closure, so $3(x+y+1)+1$ is throdd by definition.

Nice!

2. If $p|(s+t)$ and $p|s$, then $p|t$.

$p|(s+t)$ can be rewritten as $(s+t) = p \cdot x$, where $x \in \mathbb{Z}$. $p|s$ can be rewritten as $s = p \cdot y$, wher $y \in \mathbb{Z}$.

Subsitute $p \cdot y$ for s in The 1st equation

$p \cdot y + t = p \cdot x$

$t = px - py$

$t = p(x-y)$

$(x-y)$ is an integer by closure so the equation can be rewritten as

$p|t$.

So if $p|(s+t)$ and $p|s$, the $p|t$ is true.

Good

3. For any $n \in \mathbb{N}$, with $n \geq 1$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

base case $n=1$ $\sum_{i=1}^{1} i = \frac{1(1+1)}{2}$

$1 = 1$ ✓

if we assume $n=k$ to be true, then $n=k+1$ must also be true

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \quad ✓$$

$$\sum_{i=1}^{k} i + k+1 = \frac{k(k+1)}{2} + k+1$$ add $k+1$ to both sides

$$\sum_{i=1}^{k+1} i = \frac{k^2+k+2k+2}{2}$$

$$\sum_{i=1}^{k+1} i = \frac{k^2+3k+2}{2}$$

$$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$$

$$\sum_{i=1}^{k+1} i = \frac{(k+1)((k+1)+1)}{2}$$ Same form as $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ with $k+1$ in place of n ✓

since we have shown that the base case works and that if $n=k$ works, then $n=k+1$ also works, we may conclude that for any $n \in \mathbb{N}$ with $n \geq 1$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Good.

4. Determine whether a statement and its contrapositive are logically equivalent.

$P \to Q$ $\qquad$ $\neg Q \to \neg P$

P	Q	* statement $P \to Q$	$\neg Q$	$\neg P$	* contrapositive $\neg Q \to \neg P$
T	T	T ✓	F	F	T ✓
T	F	F ✓	T	F	F ✓
F	T	T ✓	F	T	T ✓
F	F	T ✓	T	T	T ✓

Since the columns for a statement and its contrapositive are identical under all circumstances, they are logically equivalent.

Good

5. $\sqrt{2}$ is irrational.

Well, s'pose $\sqrt{2}$ were rational, so $\sqrt{2} = \frac{p}{q}$, where $p, q \in \mathbb{Z}$ and share no common factors. Then squaring both sides gives $2 = \frac{p^2}{q^2}$, or $2q^2 = p^2$. Since the left side is even, p^2 is even, and by a previous result p can only be even. Then $p = 2r$ for some $r \in \mathbb{Z}$, so substituting gives $2q^2 = (2r)^2 \Rightarrow 2q^2 = 4r^2 \Rightarrow q^2 = 2r^2$. But this means q^2 is even, so q can only be even, contradicting the supposition that $\sqrt{2}$ could be written as a rational. $\square$