**Examlet 2b** **Foundations of Advanced Math** **2/24/17**

1. (a) What is $\{1,2\} \cap \{2,3\}$?

   $\{2\}$

   (b) What is $(1,2) \cap (2,3)$?

   $\emptyset$

   (c) What is $[1,2] \cap [2,3]$?

   $\{2\}$

   (d) What is $\{1,2\} \cup \{2,3\}$?

   $\{1,2,3\}$

   (e) What is $(1,2) \cup (2,3)$? 2 is not included in either interval.

   $(1,2) \cup (2,3)$

   (f) What is $[1,2] \cup [2,3]$?

   $[1,3]$

   (g) What is $\{1,2\} - \{2,3\}$?

   $\{1\}$

   (h) What is $(1,2) - (2,3)$?

   $(1,2)$

   (i) What is $[1,2] - [2,3]$? 2 is not included in the set difference.

   $[1,2)$

   (j) What is $\mathcal{P}\{1,2\}$?

   $\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$

Excellent!

2. (a) State the definition of

$$\bigcap_{i \in I} A_i$$

$\{x \mid x \in A_i \text{ for all } i \in I\}$

positive integers $\geq 2$

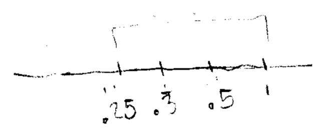
$\mathbb{Z}^+$

(b) Let $\mathbb{Z}^+ = \{n | n \in \mathbb{Z}^+ n > 0\}$. If $A_n = \left[\frac{1}{n}, 1\right] \forall n \in \mathbb{Z}^+$, what is

$[\frac{1}{2}, 1]$
$[\frac{1}{3}, 1]$
$[\frac{1}{4}, 1]$

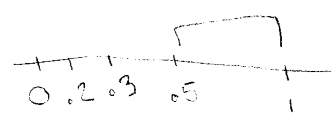
$$\bigcap_{n \in \mathbb{Z}^+} A_n$$

$\bigcap_{n \in \mathbb{Z}^+} A_n = [\frac{1}{2}, 1]$




(c) Let $\mathbb{Z}^+ = \{n | n \in \mathbb{Z}^+ n > 0\}$. If $A_n = \left[\frac{1}{n}, 1\right] \forall n \in \mathbb{Z}^+$, what is

$$\bigcup_{n \in \mathbb{Z}^+} A_n$$




$\bigcup_{n \in \mathbb{Z}^+} A_n = (0, 1]$

Great

3. $(A \cup B)' = A' \cap B'$

FIRST LETS TAKE $x \in (A \cup B)'$, THIS CAN BE WRITTEN IN LOGIC NOTATION AS $\neg(x \in A \vee x \in B)$, THROUGH DEMORGANS LAW WE KNOW THAT $\neg(x \in A \vee x \in B)$ IS LOGICALLY EQUIVALENT TO $\neg x \in A \wedge \neg x \in B$ REWRITING THIS IN SET NOTATION GIVES US $x \in A' \cap B'$, SO

$$(A \cup B)' \subseteq A' \cap B'$$

NOW LETS TAKE $x \in A' \cap B'$ THIS IMPLIES $\neg x \in A \wedge \neg x \in B$, THROUGH DEMORGANS LAW WE KNOW THAT $\neg x \in A \wedge \neg x \in B$ IS LOGICALLY EQUIVALENT TO $\neg(x \in A \vee x \in B)$ REWRITING THIS IN SET NOTATION GIVES US $x \in (A \cup B)'$, SO $A' \cap B' \subseteq (A \cup B)'$, SO BY MUTUAL INCLUSION

$$(A \cup B)' = A' \cap B' \square$$

Great

**4.**

$$A \cap \bigcup_{i\in I} B_i = \bigcup_{i\in I} (A \cap B_i)$$

Let $x \in A \cap \bigcup_{i\in I} B_i$

$x \in A$ and $x \in \bigcup_{i\in I} B_i$ by def. of intersection

$x \in A$ and $x \in B_i$ for some $i \in I$

$x \in A$ and $B_i$ for some $i \in I$

$x \in (A \cap B_i)$ for some $i \in I$

$x \in \bigcup_{i\in I} (A \cap B_i)$

So $A \cap \bigcup_{i\in I} B_i \subseteq \bigcup_{i\in I} (A \cap B_i)$

Let $x \in \bigcup_{i\in I} (A \cap B_i)$

$x \in (A \cap B_i)$ for some $i \in I$ by def. of union

$x \in A$ and $x \in B_i$ for some $i \in I$ by def. of intersection

$x \in A$ and $x \in \bigcup_{i\in I} B_i$

$x \in A \cap \bigcup_{i\in I} B_i$

So $\bigcup_{i\in I} (A \cap B_i) \subseteq A \cap \bigcup_{i\in I} B_i$

Because $A \cap \bigcup_{i\in I} B_i$ and $\bigcup_{i\in I} (A \cap B_i)$ are subsets of each other, we can say that $A \cap \bigcup_{i\in I} B_i = \bigcup_{i\in I} (A \cap B_i)$. □

Great!

5. (a) If $a > 0$ and $b > 0$, then $a + b > 0$.

$a > 0$

$a+b > b$ by C.A.P.

$a+b > b > 0$, so $a+b > 0$ by T.P.I. $\square$

Great

(b) If $a < 0$ and $b > 0$, then $a \cdot b < 0$.

$a < 0$.

$a \cdot b < 0 \cdot b$ by C.M.P, since $b > 0$.

$\therefore$ $a \cdot b < 0$

Excellent!