Examlet 4a **Foundations of Advanced Math** **4/14/17**

1. (a) State the definition of a relation from A to B.

A relation from A to B is a subset of $A \times B$.

(b) Give an example of a relation from $\{1, 2, 3, 4\}$ to $\{1, 2, 3, 4\}$ which is not reflexive or symmetric, but is transitive.

$\{(1, 2)\}$ is vacuously transitive, we don't know $2 \sim 1$ so it's not symmetric & we don't know $(1,1)$ or $(2,2)$ so it's not reflexive

Good

A set of Pair wise disjoint Sets whose union is all of S

2. Which of the following are partitions of $S = \{a, b, c, d, e\}$? Mark all which are.

- [x] $\{\{a, b\}, \{c, d, e\}\}$ Yes because it is a set of Sets, and the union is all of S.
- [ ] $\{\{a\}, \{c\}\}$ No because the union is not all of S.
- [ ] $\{a, b, c, d, e\}$ No because this is not a set of Sets.
- [ ] $\{a, b, c, d\}, \{e\}$ No because this is not a set of sets.
- [x] $\{\{a, b, c, d\}, \{e\}\}$ Yes because this is a set of Pairwise disjoint sets whos union is all of S.

Excellent!

3. Express the definition of a surjective function in terms of ordered pairs.

f is surjective iff $\forall b \in B$ $\exists a \in A$ such that $(a,b) \in f$

Good

4. Let S be a set and Π a partition of S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then Π is a reflexive relation.

Let $a \in S$. Π is a partition of S so the union of Π is all of S. So $\exists P \in \Pi$ for which $a \in P$. Since $a \in P$ and $a \in P$ then $a \sim a$ and it is a reflexive relation

Nice!

5. (a) State the definition of a graph.

A graph is a set of vertices, along with a set of edges, where an edge is a set of exactly two distinct vertices.

Great

(b) Suppose G is a graph with every vertex having degree at least 1. Create a relation $\sim$ on the vertices of G by saying that two vertices v_1, v_2 of G are related iff there exists a walk from v_1 to v_2 which has no edge used more than once. Is $\sim$ reflexive? Symmetric? Transitive?

Reflexive:

$\forall v \in G$, $\exists$ a walk between v and v with no edges. So $\forall v \in G$, $v \sim v$, and it is reflexive. Good

Symmetric:

$\forall v_1 \sim v_2$, $\exists$ a walk with no repeated edges between V_1 and V_2. So if we traverse that walk in reverse, we can see similarly a walk with no repeated edges exists from V_2 to V_1. So $V_1 \sim V_2 \Longleftrightarrow V_2 \sim V_1$, and it is symmetric. Great

Transitive:

$\forall V_1 \sim V_2, V_2 \sim V_3$, $\exists$ a walk from V_1 to V_2 and a walk from V_2 to V_3 with no repeated edges. Suppose we create a new walk, starting at V_1 towards V_2. As we construct this walk, every time we add a vertex we can check if that vertex exists in the walk from V_2 to V_3. If not, we add another edge, and check the next vertex. If it is in the walk from V_2 to V_3, we then use that walk from that vertex on and proceed to V_3. Because we check every vertex we added to our walk to see if it was in the walk from V_2 to V_3, we know that no vertices will be repeated in the walk we construct from V_1 to V_2, and therefore no edges also. Since we constructed a walk from V_1 to V_3 with no repeated edges, $V_1 \sim V_2 \wedge V_2 \sim V_3 \Rightarrow V_1 \sim V_3$, and it is transitive.

Nice!