Examlet 4b **Foundations of Advanced Math** **4/14/17**

1. (a) State the definition of a relation from A to B.

a relation from A to B is a subset of $A \times B$

Good

(b) Give an example of a relation from $\{1, 2, 3, 4\}$ to $\{1, 2, 3, 4\}$ which is reflexive and symmetric, but is not transitive.

$\{(1,1), (1,2), (2,3), (2,1), (3,2), (2,2), (3,3), (4,4)\}$

- reflexive because every element is related to itself
- symmetric b/c every $a \sim b$ is $b \sim a$
- not transitive because $1 \sim 2 \wedge 2 \sim 3$ but $1 \not\sim 3$

Good

2. Which of the following are partitions of $S = \{a, b, c, d, e\}$? Mark all which are.

- ☑ $\{\{a, b, c\}, \{d, e\}\}$
- ☐ $\{\{a, b, d\}, \{c\}\}$ - no b/c there is no e so union of all partitions is not S
- ☐ $\{\{a, b\}, \{c, d, e\}, \cancel{\emptyset}\}$ partitions are pairwise disjoint non-empty subsets of S
- ☐ $\{a, c, d\}, \{b, e\}$ - needs other set of { }
- ☐ $\{\{a, b\}, \{b, c\}, \{c, d\}, \{e\}\}$ - not pairwise disjoint

Excellent!

3. Express the definition of a surjective function in terms of ordered pairs.

f is surjective iff $\forall b \in B$ $\exists a \in A$ such that $(a, b) \in f$

Good.

4. Let S be a set and Π a partition of S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$. Then Π is a reflexive relation.

Let $a \in S$. Π is a partition of S so the Union of Π is all of S. So $\exists P \in \Pi$ for which $a \in P$. Since $a \in P$ and $a \in P$ then $a \sim a$ and it is a reflexive relation

Nice!

5. (a) State the definition of a graph.

A graph is a set of vertices, along with a set of edges, where an edge is a set of exactly two distinct vertices.

Great

(b) Suppose G is a graph with every vertex having degree at least 1. Create a relation $\sim$ on the vertices of G by saying that two vertices v_1, v_2 of G are related iff there exists a walk from v_1 to v_2 which has no edge used more than once. Is $\sim$ reflexive? Symmetric? Transitive?

Reflexive:

$\forall v \in G$, $\exists$ a walk between v and v with no edges. So $\forall v \in G$, $v \sim v$, and it is reflexive. Good

Symmetric:

$\forall v_1 \sim v_2$, $\exists$ a walk with no repeated edges between v_1 and v_2. So if we traverse that walk in reverse, we can see similarly a walk with no repeated edges exists from v_2 to v_1. So $v_1 \sim v_2 \Longleftrightarrow v_2 \sim v_1$, and it is symmetric. Great

Transitive:

$\forall v_1 \sim v_2, v_2 \sim v_3$, $\exists$ a walk from v_1 to v_2 and a walk from v_2 to v_3 with no repeated edges. Suppose we create a new walk, starting at v_1 towards v_2. As we construct this walk, every time we add a vertex we can check if that vertex exists in the walk from v_2 to v_3. If not, we add another edge, and check the next vertex. If it is in the walk from v_2 to v_3, we then use that walk from that vertex on and proceed to v_3. Because we check every vertex we added to our walk to see if it was in the walk from v_2 to v_3, we know that no vertices will be repeated in the walk we construct from v_1 to v_2, and therefore no edges also. Since we constructed a walk from v_1 to v_3 with no repeated edges, $v_1 \sim v_2 \wedge v_2 \sim v_3 \Rightarrow v_1 \sim v_3$, and it is transitive.

Nice!