Examlet 1 **Advanced Geometry** **2/6/17**

1. a) State the definition of a line segment.

Let A and B be points. Line segment $\overline{AB}$ is the following set:

$$\overline{AB} = \{A, B\} \cup \{P \mid A*P*B\}$$

Where P is a point.

Good.

b) State the definition of an angle bisector.

Let $\angle ABC$ be an angle and let D be a point in the interior of that angle. The ray $\overrightarrow{BD}$ is an angle bisector iff $\mu(\angle ABD) = \mu(\angle DBC)$.

Great

c) State the definition of congruent triangles.

Triangles $\triangle ABC$ and $\triangle DEF$ are congruent iff all of their corresponding angles and line segments are congruent.

Great

2. Name and state the three different parallel postulates we have discussed.

Euclidean Parallel Postulate: For any line ℓ and any point P not on ℓ, there exists exactly one line m such that P lies on m and $m \parallel \ell$.

Elliptical Parallel Postulate: For any line ℓ and any point P not on ℓ, there exists no line m such that P lies on m and $m \parallel \ell$.

Hyperbolic Parallel Postulate: For any line ℓ and any point P not on ℓ, there exist at least two lines m and n such that P lies on both m and n, and $m \parallel \ell$ and $n \parallel \ell$.

Nice!

3. Recall the three axioms of Incidence Geometry:

- **Incidence Axiom 1:** For every pair of distinct points P and Q there exists exactly one line l such that both P and Q lie on l.

- **Incidence Axiom 2:** For every line l there exist at least two distinct points P and Q such that both P and Q lie on l.

- **Incidence Axiom 3:** There exist three points that do not all lie on any one line.

a) Give an example of a geometry satisfying Incidence Axioms 1 and 3 but not 2.

- Let $\{A, B, C\}$ be a set of points. Let $\{A, B\}$, $\{A\}$, $\{A, C\}$, $\{B, C\}$ be defined as lines
- Axioms 1 and 3 are satisfied since A, B, C do not lie on any one line and for every pair of points $\{A, B, C\}$ there is exactly one line.
- Axiom 2 does not meet since if you pick the line $\{A\}$ there are not at least two points that lie on that line

Great

b) Give an example of a geometry satisfying Incidence Axioms 2 and 3 but not 1.

- Let $\{A, B, C, D\}$ be a set of points. And $\{A, B\}$, $\{B, C\}$ and $\{A, C\}$ all be lines.
- Thus Incidence Axiom 2 and 3 are satisfied since for every line there are at least two points. And points, A, B, C do not all lie on one line.
- Axiom 1 is not satisfied since if you pick point A and D there is no line that those points lie on.

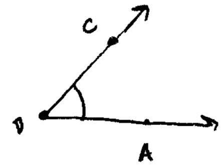
Excellent

Use Definition

~~Need to show $\angle ABC \cong \angle CBA$~~

4. Show that $\angle ABC = \angle CBA$.

- Let A, B, C be three distinct, noncollinear points s.t. $\overrightarrow{BC}$ and $\overrightarrow{BA}$ are nonopposite rays
- Let $\angle ABC$ be the union of two rays $\overrightarrow{BC}$ and $\overrightarrow{BA}$. (Definition of Angle)



- So ~~Since~~ $\angle ABC = \overrightarrow{BC} \cup \overrightarrow{BA}$
- We also know that $\angle CBA$ is the union of two rays $\overrightarrow{BC}$ and $\overrightarrow{BA}$ (definition of Angle)
- So $\angle CBA = \overrightarrow{BC} \cup \overrightarrow{BA}$
- Thus, $\angle ABC = \overrightarrow{BC} \cup \overrightarrow{BA} = \angle CBA$
- $\therefore$ $\angle ABC = \angle CBA$

Yes.

5. Prove that if D and E are two distinct points, then there exists a unique perpendicular bisector for $\overline{DE}$.

By previous theorm, we know that for any point P on a line ℓ, there $\exists$ one line m such that P lies on m & $\ell \perp m$. Now, considering the line segment $\overline{DE}$, we know by the uniqueness & existence of a midpoint for a line segment that $\exists$ a point F such that $\overline{DF} + \overline{FE} = \overline{DE}$. Since this point will lie on exactly one line perpendicular to $\overline{DE}$, & since F is a unique point, $\exists$ a unique perpendicular bisector for $\overline{DE}$.

Great