

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Evaluate

$$\int \frac{2+x^2}{1+x^2} dx$$

2. Evaluate

$$\int x e^x dx$$

3. Evaluate

$$\int x^2 \sqrt{x^3 - 8} dx$$

4. Evaluate

$$\int_3^{\infty} \frac{1}{(x-2)^{3/2}} dx$$

5. Evaluate

$$\int \sec^4 \theta \tan \theta d\theta$$

6. Evaluate

$$\int_0^1 \frac{2}{2x^2 + 3x + 1} dx$$

7. Biff is a Calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, Calc is tough! I thought I had it all figured out, but I guess it's just too much for me. We had this assignment and I, like, outsourced it to Mathematica, right? So for this one where we were supposed to integrate 1 over  $3x-2$ , Mathematica said  $\frac{1}{3} \ln(3x - 2)$ , so I wrote that down. But the grader took off points and wrote this nasty note about something general and some domain thing, and about how even a computer could do as well as I did, like that was a bad thing. But dude, I think computers are automatically right, right?"

Help Biff out by explaining what shortcomings there might be to his answer, and how he should improve it.

8. Derive the reduction formula

$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

9. It turns out there's a reason to care about  $\int_{-r}^r \frac{r}{\sqrt{r^2-x^2}} dx$ . Find the value of this integral.

10. Derive Line 23 from the Table of Integrals:

$$\int \frac{\sqrt{a^2 + u^2}}{u} du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$$

Extra Credit [5 points possible]: Think about what happens when you add up numbers of the form  $\frac{1}{n(n+1)}$ , for integer values of  $n$  that start with 1 and continue upward to some value  $k$ . What can you say about the result?