Examlet 4 **Foundations of Advanced Math** **4/16/18**

1. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $a \sim b \Leftrightarrow 3|(b-a)$.

   (a) Determine whether $\sim$ is reflexive.

   (b) Determine whether $\sim$ is symmetric.

   (c) Determine whether $\sim$ is transitive.

2. Consider the relation on $\mathbb{N}$ defined by $a \approx b \Leftrightarrow b = n \cdot a$ for some $n \in \mathbb{N}$.

   (a) Determine whether $\approx$ is reflexive.

   (b) Determine whether $\approx$ is symmetric.

   (c) Determine whether $\approx$ is transitive.

3. Consider the relation $R = \{(1,1),(1,2),(2,1),(2,2),(3,3),(3,5),(4,4),(5,3),(5,5)\}$. Give the equivalence classes of R and the partition associated with R.

4. We say that two vertices v_1 and v_2 of a graph G are **on a common cycle of** $G \Leftrightarrow \exists$ a cycle including v_1 and v_2.

   (a) The relation of being on a common cycle of a graph is reflexive.

   (b) The relation of being on a common cycle of a graph is symmetric.

   (c) The relation of being on a common cycle of a graph is transitive.

5. The number of edges in a tree with n vertices is _____.
   [Yes, you need to justify your answer.]