Problem Set 3 **Foundations** **Due 1/29/18**

Four of these problems will be graded (our choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. $\sqrt{6}$ is irrational.
2. The product of a non-zero rational and an irrational number is irrational.
3. Critique the following proof:
   Proposition: If $2 \mid p^2$ then $2|p$.
   Proof: Assume $2 \nmid p$, i.e. $p \neq 2n, \exists n \in \mathbb{Z}$. So p must be odd. $\forall m \in \mathbb{Z}, p = 2m + 1$. Now $p^2 = (2m+1)^2 = 4m^2 + 4m + 1$ is odd because of the +1 and not even, so $2 \nmid p^2$.
4. Consider the formula $1 + 2 + 3 + ... + n = \frac{n^2+n+1}{2}$.
   (a) Write the formula in sigma notation.
   (b) Show that if this formula works for $n = k$, then it also must work for $n = k + 1$.
   (c) Explain why mathematical induction does not prove that this formula is true for all $n \in \mathbb{N}$.
5. For any $n \in \mathbb{Z}^+$,
$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

The Fibonacci numbers are defined by the two-term recurrence relationship

$$F_1 = 1, \;\; F_2 = 1, \;\; F_{n+2} = F_{n+1} + F_n \;\text{ for }\; n \in \mathbb{Z}^+$$

6. Find F_3 through F_{10}.
7. $F_1 + F_2 + ... + F_n = F_{n+2} - 1$ for every $n \in \mathbb{Z}^+$.
8. $F_1 + F_3 + ... + F_{2n-1} = F_{2n}$ for every $n \in \mathbb{Z}^+$.