Midterm Exam A **Set Theory & Topology** **3/2/18**

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of those you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Dont panic.

1. State the definition of a topology.

2. (a) State the (topological) definition of continuity.

(b) Give an example of a function which is $\mathscr{U}-\mathscr{U}$ continuous but not $\mathscr{C}-\mathscr{C}$ continuous, or explain why it can't be done.

3. (a) State the (topological) definition of a closed set.

(b) Give three examples of sets which are both open and closed in $(\mathbb{R}, \mathscr{H})$.

4. (a) State the definition of a homeomorphism.

(b) Show that in $(\mathbb{R}, \mathscr{U})$, the interval (a, b) is homeomorphic to $\mathbb{R}$.

5. Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces. If A and B are closed subsets of X and Y, respectively, then $A \times B$ is a closed subset of $X \times Y$.

6. Show that the composition of homeomorphisms is a homeomorphism. Feel free to note the portions that were taken care of in Foundations, but provide details on those that were not.

7. Let $\Lambda = \mathbb{Z}^+$ and for each $i \in \Lambda$, let $X_i = \mathbb{R}$ and let $\mathscr{T}_i = \mathscr{U}$. Which of the following are closed subsets of the product space $\times\{X_i : i \in \Lambda\}$? Support your answer well.

(a) $\times\{U_i : i \in \Lambda\}$, where $U_i = [0, 1]$ for each $i \in \Lambda$.

(b) $\times\{U_i : i \in \Lambda\}$, where $U_i = [0, 1]$ if i is an odd integer and $\mathbb{R}$ if i is an even integer.

(c) $\times\{U_i : i \in \Lambda\}$, where $U_1 = [0, 1]$ and $U_i = \mathbb{R}$ otherwise.

8. The space $(\mathbb{R}, \mathscr{U})$ is connected.

□ A. Let $(X_1, \mathscr{T}_1), (X_2, \mathscr{T}_2), ..., (X_n, \mathscr{T}_n)$ be topological spaces. Let F_i be a closed subset of X_i for each $i \in \{1, 2, ..., n\}$. Prove that $F_1 \times F_2 \times ... \times F_n$ is a closed subset of the product space $X_1 \times X_2 \times ... \times X_n$.

☐ B. If $\{(X_\alpha, \mathscr{T}_\alpha) : \alpha \in \Lambda\}$ is a collection of topological spaces, Λ is infinite, and $A_\alpha \subseteq X_\alpha$ for each $\alpha \in \Lambda$, then $\mathrm{Int}(\times\{A_\alpha : \alpha \in \Lambda\}) = \times\{\mathrm{Int}(A_\alpha) : \alpha \in \Lambda\}$.

□ C. Is the collection of intervals of the form $(x, x+2)$ for $x \in \mathbb{R}$ a base for a topology?

□ D. For sets A and B in a topological space, $\mathrm{Cl}(A) \cap \mathrm{Cl}(B) = \mathrm{Cl}(A \cap B)$.

☐ E. Let $(X, \mathscr{T})$ be a topological space and let $A \subseteq X$. Then A is open iff $A = \text{Int}(A)$.

□ F. $(\mathbb{R}, \mathscr{U})$ is homeomorphic to the unit circle.