Problem Set 2 **Set Theory & Topology** **Due 1/26/18**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Four of these problems will be selected (by Jon) for grading, with each worth 5 points.

1. [Baker 2.2.10] Show that the collection $\mathscr{C}$ given in Example 2.2.3 is a topology for $\mathbb{R}$.

2. [Baker 2.2.12] Determine if the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} 2 & \text{if } x > 1 \\ -2 & \text{if } x \leq 1 \end{cases}$$

   is

   (a) $\mathscr{U} - \mathscr{U}$ continuous
   (b) $\mathscr{U} - \mathscr{H}$ continuous
   (c) $\mathscr{U} - \mathscr{C}$ continuous
   (d) $\mathscr{H} - \mathscr{U}$ continuous
   (e) $\mathscr{H} - \mathscr{H}$ continuous
   (f) $\mathscr{C} - \mathscr{H}$ continuous
   (g) $\mathscr{C} - \mathscr{C}$ continuous

3. [Baker 2.3.14] Let A and B be subsets of a topolpogical space $(X, \mathscr{T})$. Show that $(X - \mathrm{Cl}(A)) \cup (X - \mathrm{Cl}(B)) \subseteq X - \mathrm{Cl}(A \cap B)$. Find an example that shows these sets are not in general equal.

4. [Baker 2.3.15] Let A and B be subsets of a topological space $(X, \mathscr{T})$. Show that

$$X - \mathrm{Cl}(A \cup B) = (X - \mathrm{Cl}(A)) \cap (X - \mathrm{Cl}(B)).$$

5. [Baker 2.4.6] Let $X = \{a, b, c\}$ and $\mathscr{T} = \{X, \varnothing, \{a, b\}, \{a, c\}, \{a\}\}$. Let $A = \{a, c\}$. Find each of the following sets:

   (a) $\mathrm{Cl}(A)$
   (b) $\mathrm{Int}(A)$
   (c) $\mathrm{Bd}(A)$

(d) Ext(A)

6. [Baker 2.4.7] Let $X = \mathbb{R}$ and $\mathscr{T} = \{U \subseteq X : 2 \in X \text{ or } U = \emptyset\}$. Let $A = \{1, 3\}$. Find each of the following sets:

    (a) Cl(A)

    (b) Int(A)

    (c) Bd(A)

    (d) Ext(A)

7. [Baker 2.4.13] Prove Theorem 2.4.21, i.e. Let $(X, \mathscr{T})$ be a topological space and let $A \subseteq X$. Then the sets Int(A), Bd(A), and Ext(A) are pairwise disjoint and $X = \text{Int}(A) \cup \text{Bd}(A) \cup \text{Ext}(A)$.