Reference Sheet: Series Tests   Calculus 2   3/26/2019

The Test for Divergence: If $\sum a_n$ is a series for which $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ diverges.

The Geometric Series Test: If a series is of the form $\sum_{n=1}^{\infty} a \cdot r^{n-1}$, then the series converges $\left(\text{to } \frac{a}{1-r}\right)$ if and only if $|r| < 1$.

The Integral Test: Suppose f(x) is a continuous, positive, decreasing function on [c, ∞) for some $c \geq 0$, with $a_n = f(n)$ for all n:

- If $\int_c^{\infty} f(x)\,dx$ converges, then Σa_n converges also.
- If $\int_c^{\infty} f(x)\,dx$ diverges, then Σa_n diverges also.

The Comparison Test: If Σa_n and Σb_n are both series with their terms all positive, and:

- $a_n \leq b_n$ with Σb_n convergent, then Σa_n converges also.
- $a_n \geq b_n$ with Σb_n divergent, then Σa_n diverges also.

The Limit Comparison Test: If Σa_n and Σb_n are both series with their terms all positive, and

$$\lim_{n\to\infty} \frac{a_n}{b_n} = L$$

for some finite, positive number L, then either both series converge or both series diverge.

The Ratio Test: If Σa_n is a series for which

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$$

then:

- If $L < 1$ then the series converges absolutely.
- If $L > 1$ (or if the limit diverges to $+\infty$) then the series diverges.

The Alternating Series Test: If $\Sigma(-1)^{n+1} a_n$, with $a_n \geq 0$ for all n, is a series for which

- the sequence $\{a_n\}$ tends to zero, i.e. $\lim_{n\to\infty} a_n = 0$
- the sequence $\{a_n\}$ is decreasing, i.e. $a_{n+1} \leq a_n$ for all n

then the series converges.