

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. The sum of two odd integers must be even.
2. If  $p, q, r \in \mathbb{Z}$  with  $p, q$ , and  $r$  odd, then  $p + q + r$  is odd.
3. If  $p, q, r \in \mathbb{Z}$  with  $p + q + r$  odd, then  $p, q$ , and  $r$  are odd.
4. The product of an even and an odd integer is even.
5. The sum of a throddodd and a throdd integer is threven.
6. The product of a throdd and a throdd integer is throdd.
7. The sum of any three consecutive integers is threven.
8. Critique the following proof of the proposition “The sum of two odd integers must be even”:

Well, let  $m$  and  $n$  be odd integers, so  $m = 2a + 1$  and  $n = 2a + 1$ , where  $a \in \mathbb{Z}$ . Then  $m + n = (2a + 1) + (2a + 1) = 4a + 2 = 2(2a + 1)$ , and  $2a + 1$  is an integer by closure, so  $m + n$  is 2 times an integer and thus even by definition.  $\square$