Problem Set 4 **Foundations** **Due 2/18/19**

Four of these problems will be graded (our choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Show that 53 divides every number in the sequence
$$1007, 10017, 100117, 1001117, \ldots$$
2. We need to put n cents of stamps on an envelope, but we only have (an unlimited supply of) 5-cent and 12-cent stamps. Prove that we can perform the task if $n \geq 44$.
3. Prove that if A has n elements, then $\mathcal{P}(A)$ has 2^n elements. [Hint: Induction!]
4. For any sets A and B, $(A \bigcup B)' = A' \bigcap B'$.
5. For any sets A, B, and C, $A \bigcup (B \bigcap C) = (A \bigcup B) \bigcap (A \bigcup C)$.
6. Score at least 7 out of 8 on the Set Theory Gateway on WeBWorK.
7. Suppose $A \subseteq B$ and $C \subseteq D$. Show that $A \cap C \subseteq B \cap D$.
8. Show that $A - (B \cap A) = (A - B) \cap A$.
9. Show that
$$\left(\bigcap_{i \in I} A_i \right)' = \bigcup_{i \in I} A_i'$$
10. Show that
$$A \cup \bigcap_{i \in I} B_i = \bigcap_{i \in I} (A \cup B_i)$$