Examlet 3 **Advanced Geometry** **4/12/19**

1. a) State the Neutral Area Postulate.

associated with any polygonal region R is a nonnegative number $\alpha(R)$ that is the area of R such that the following conditions are satisfied

Congruence

for any two congruent triangles, $\triangle ABC \cong \triangle DEF$

Their areas are equal, $\alpha(\triangle ABC) = \alpha(\triangle DEF)$

Additivity

for any two nonoverlapping polygonal regions R_1 and R_2 whose union is R, $\alpha(R) = \alpha(R_1) + \alpha(R_2)$

Good

b) State the Euclidean Area Postulate.

for any rectangle $\square ABCD$

$\alpha(\square ABCD) = AB \cdot BC$

Great

$\beta, B \quad \gamma$

2. A triangle has $\alpha = 36°$, $A = 6.0$, and $C = 10.0$. Solve for the remaining measurements, accurate to the nearest tenth.

$$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$

$$C^2 = A^2 + B^2 - 2AB \cdot \cos\gamma$$

$$\frac{\sin 36°}{6.0} = \frac{\sin\gamma}{10.0}$$

$\sin\gamma = 0.9796$

$\boxed{\gamma = 78.4°}$

$\alpha + \beta + \gamma = 180°$

$36° + \beta + 78.4° = 180$

$\beta + 114.4 = 180$

$\boxed{\beta = 65.6°}$

$$\frac{\sin 36°}{6.0} = \frac{\sin 65.6°}{B}$$

$\boxed{B = 9.3}$

Excellent!

Or -

$\boxed{\gamma = 101.6°}$

$\alpha + \beta + \gamma = 180$

$36 + \beta + 101.6 = 180$

$\boxed{\beta = 42.4°}$

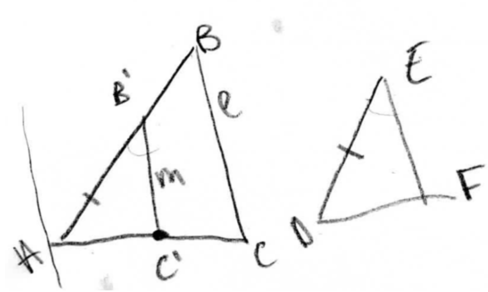
$$\frac{\sin 36°}{6.0} = \frac{\sin 42.4°}{B}$$

$\boxed{B = 6.9}$

3. Provide good justifications in the blanks below for the corresponding statements:
Proposition: If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\triangle ABC \sim \triangle DEF$, then

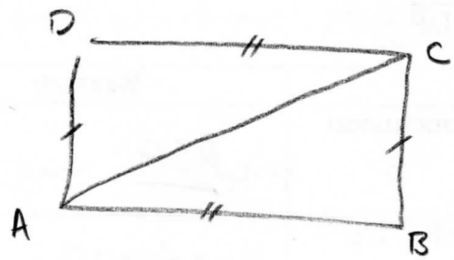
$$\frac{AB}{AC} = \frac{DE}{DF}$$

Statement:	**Reason:**
If $AB = DE$, then $\triangle ABC \cong \triangle DEF$ and the conclusion is evident.	ASA
So suppose $AB \neq DE$. Either $AB > DE$ or $AB < DE$.	trichotomy
Change notation, if necessary, so that $AB > DE$. Choose a point B' on $\overline{AB}$ such that $AB' = DE$.	Existence postulate/ Ruler post
Let m be the line through B' such that m is parallel to $\ell = \overleftrightarrow{BC}$	Existence of parallel lines
and let C' be the point at which m intersects $\overline{AC}$.	Pasch's axiom
Then $\angle AB'C' \cong \angle DEF$ $\angle ABC \cong \angle DEF$	Conv. to the alt. interior angles theorem
Then $\triangle AB'C' \cong \triangle DEF$	ASA
Let n be the line through A that is parallel to ℓ and m.	Existence of parallel lines
Then $AB'/AB = AC'/AC$ and so $DE/AB = DF/AC$.	parallel projection theorem
$DE/DF = AB/AC$ as desired.	Algebra



Excellent

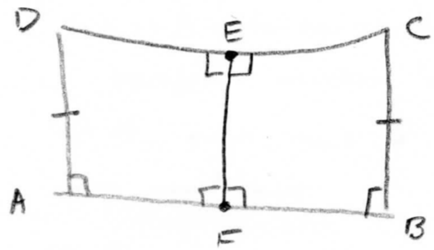
4. Prove that if $\square ABCD$ is a parallelogram in the Euclidean plane and diagonal $\overline{AC}$ divides the quadrilateral into congruent triangles, then the opposite sides are congruent.



Let $\square ABCD$ be a parallelogram such that $AB \parallel DC$ and $AD \parallel BC$. Construct diagonal AC. Since AC acts as a transversal for AB and DC, then by the Converse of Alt. Int Angles $\angle BAC \cong \angle DCA$. Also, since AC acts as a transversal for AD and BC, then by converse of Alt Int Angles $\angle DAC \cong \angle BCA$. And since $\triangle ACB$ and $\triangle CAD$ have a common side AC, then $\triangle ACB \cong \triangle CAD$ by ASA. Therefore, by the definition of congruent triangles $AD \cong BC$ and $AB \cong DC$, thus proving that the opposite sides are congruent. $\square$

Excellent

5. Prove that for a Saccheri quadrilateral in the hyperbolic plane, the length of the summit must be greater than the length of the base.



Must prove that for a Saccheri quad in a hyperbolic plane, the length of the summit must be greater than the base.

Proof: Let □ABCD be a quadrilateral such that it is a Saccheri quadrilateral. Construct midpoints on the summit (DC) and the base (AB) and call these points E and F respectively. Now construct segment EF. By previous theorem we know that for a saccheri quad the segment EF is a perp. line for both DC and AB. Therefore this creates Lambert quadrilaterals □AFED and □EFBC. Since we are in a hyperbolic plane then ∠C and ∠D are acute because rectangles do not exist. But we know that for a lambert quad, that the length of segments enbetween two right angles are strictly less than its opposite side. So this implies AF < DE and FB < EC. Therefore by Algebra we get AF + FB < DE + EC and since AF + FB is the base and DE + EC is the summit, we just proved that for a Saccheri quad in the hyperbolic plane the summit has a greater length than the base. □

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Excellent