Examlet 1 Advanced Geometry 2/11/19

1. Let $A = (0, 0)$ and $B = (2, 1)$, and let ρ be the taxicab metric.
   a) Describe all points on $\overline{AB}$.

   b) Describe all points on $\overrightarrow{AB}$.

2. a) State the SAS Postulate.

   b) Explain clearly why the SAS Postulate is important as an axiom for Neutral Geometry.

3. Recall the three axioms of Incidence Geometry:

- **Incidence Axiom 1:** For every pair of distinct points P and Q there exists exactly one line ℓ such that both P and Q lie on ℓ.
- **Incidence Axiom 2:** For every line ℓ there exist at least two distinct points P and Q such that both P and Q lie on ℓ.
- **Incidence Axiom 3:** There exist three points that do not all lie on any one line.

a) Consider a regular tetrahedron, with the faces regarded as lines and the vertices regarded as points. Which of the incidence axioms does this satisfy, and why?

b) Consider a cube, with the faces regarded as lines and the vertices regarded as points. Which of the incidence axioms does this satisfy, and why?

c) Consider a regular dodecahedron, with the faces regarded as lines and the vertices regarded as points. Which of the incidence axioms does this satisfy, and why?

4. a) Consider a regular tetrahedron, with the faces regarded as lines and the vertices regarded as points. Which parallel postulates, if any, does it satisfy? Why?

b) Consider a cube, with the faces regarded as lines and the vertices regarded as points. Which parallel postulates, if any, does it satisfy? Why?

c) Consider a regular dodecahedron, with the faces regarded as lines and the vertices regarded as points. Which parallel postulates, if any, does it satisfy? Why?

5. Provide good justifications in the blanks below for the corresponding statements:

Theorem: If $\triangle ABC$ is a triangle and $\overline{AB} \cong \overline{AC}$, then $\angle ABC \cong \angle ACB$

Statement	**Reason**
Let $\triangle ABC$ be a triangle such that $\overline{AB} \cong \overline{AC}$.	Given
We must prove that $\angle ABC \cong \angle ACB$.	Because that's what we must prove.
Let D be the point in the interior of $\angle BAC$ such that $\overrightarrow{AD}$ is the bisector of $\angle BAC$.	
There is a point E at which the ray $\overrightarrow{AD}$ intersects the segment $\overline{BC}$.	
Then $\triangle BAE \cong \triangle CAE$	
and so $\angle ABE \cong \angle ACE$.	
This completes the proof because $\angle ACE = \angle ACB$ and $\angle ABE = \angle ABC$. $\square$	