Examlet 2 Advanced Geometry 3/13/19

1. a) State the definition of a *scalene triangle*.

a triangle is Scalene iff all three edges have different lengths.

b) State the definition of a *quadrilateral*.

Let A, B, C, D be four distinct points, no three of which are colinear and let $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, & $\overline{DA}$ have no points in common except for the endpoints. Then the union of the 4 segments is called a quadrilateral

Good

c) State the Saccheri-Legendre Theorem.

for any triangle (call this $\triangle ABC$)

$\sigma(\triangle ABC) \leq 180°$

d) State the Scalene Inequality.

for any triangle the greater side lies opposite the greater angle and the greater angle lies opposite the greater side.

Great

e) State the Universal Hyperbolic Theorem.

If there exists one line l_o, an external point P_o, and at least two lines passing through P_o parallel to l_o then for any line l and any external point P, there exist at least two lines that pass through P that are parallel to l.

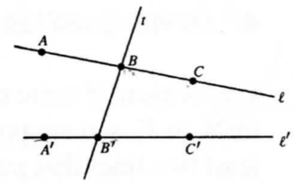
Great

2. Which of the following are equivalent (given the other postulates of neutral geometry) to the Euclidean Parallel Postulate? Check all that apply.

- [ ] The double perpendicular construction
- [ ] The Saccheri-Legendre Theorem
- [x] Existence of rectangles
- [x] Euclid's Postulate V
- [x] Converse of the Alternate Interior Angles Theorem
- [x] If $\triangle ABC$ is a triangle, then $\sigma(\triangle ABC) = 180°$.
- [x] Clairaut's Axiom
- [ ] A petunia ate my unicorn.

  If it ate all but one parallel line for any external point P to a line ℓ, maybe.
- [x] There exists a triangle whose defect is $0°$.
- [ ] The Universal Hyperbolic Theorem

3. Provide good justifications in the blanks below for the corresponding statements:



Proposition: If ℓ and ℓ' are two lines cut by a transversal t in such a way that a pair of alternate interior angles is congruent, then ℓ is parallel to ℓ'.

Statement:	Reason:
Let ℓ and ℓ' be two lines cut by transversal t such that a pair of alternate interior angles is congruent.	Hypothesis
Choose points A, B, C, and A', B', C' as in the figure above. Suppose $\angle A'B'B \cong \angle B'BC$.	Point construction Hypothesis
We must prove that ℓ is parallel to ℓ'. Suppose there exists a point D such that D lies on both ℓ and ℓ'.	RAA
If D lies on the same side of t as C, then $\angle A'B'B$ is an exterior angle for $\triangle BB'D$,	def of exterior angle
while $\angle B'BC$ is a remote interior angle for $\triangle BB'D$.	def of remote interior angle
This is a contradiction.	exterior angle theorem
In case D lies on the same side of t as A, then $\angle B'BC$ is an exterior angle and $\angle A'B'B$ is a remote interior angle for $\triangle BB'D$,	def exterior angle def remote interior angle
and again we have a contradiction.	exterior angle theorem
Since D must lie on one of the two sides of t,	plane seperation post
we are forced to conclude that the proposition holds.	def of parallel

Great

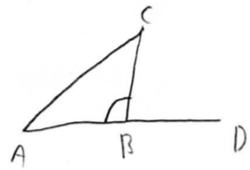
4. Provide good justifications in the blanks below for the corresponding statements:

Proposition: If there exists one line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0, then for every line ℓ and for every external point P there exist at least two lines that pass through P and are parallel to ℓ.

Statement:	Reason:
S'pose there exists a line ℓ_0, an external point P_0, and at least two lines that pass through P_0 and are parallel to ℓ_0.	Hypothesis
Then the Euclidean Parallel Postulate fails.	Euclidean Parallel Postulate
No rectangle exists.	Clairouts Axiom is equivalent to the Euclidean Parallel Postulate
Let ℓ be a line and P an external point.	Hypothesis
We must prove that there are at least two lines through P that are both parallel to ℓ. Drop a perpendicular to ℓ through P and call the foot of that perpendicular Q.	Existence + Uniqueness of Perpendiculars
Let m be the line through P that is perpendicular to $\overleftrightarrow{PQ}$.	Existence & Uniqueness of Perpindiculars
Choose a point R on ℓ that is different from Q and let t be the line through R that is perpendicular to ℓ.	Existence + Uniqueness of Perpendiculars
Drop a perpendicular from P to t and call the foot of the perpendicular S.	Existence + Uniqueness of Perpindiculars
Now $\square PQRS$ is a Lambert quadrilateral.	Def^{n} of Lambert Quadrilateral
But $\square PQRS$ is not a rectangle (reason?), so $\angle QPS$ is not a right angle and $\overleftrightarrow{PS} \neq m$.	Already said no rectangle exist
Nevertheless $\overleftrightarrow{PS}$ is parallel to ℓ,	Alternate interior Angles Theorem
so our proof is complete.	Because our proof is complete.

Excellent!

5. a) Prove or give a counterexample: If one interior angle of a triangle is obtuse, then both the other interior angles are acute.



LET ABC BE A TRIANGLE, THEN TAKE A POINT D SUCH THAT $A*B*D$ AND $\angle ABC$ AND $\angle DBC$ ARE A LINEAR PAIR. THEN SINCE $\mu(\angle ABC) + \mu(\angle DBC) = 180$ AND $\mu(\angle ABC) > 90$ THEN $\mu(\angle DBC) < 90$ AND SINCE DBC IS AN EXTERIOR ANGLE FOR ΔABC THE OTHER INTERIOR ANGLES $\angle BAC$, $\angle BCA < 90$ AND THUS ACUTE $\square$

Nice

b) Prove or give a counterexample: If one interior angle of a triangle is acute, then at least one of the other interior angles is obtuse.

FALSE, TAKE ALL ANGLES AS 60° THEN ALL ANGLES ARE ACUTE, SO NONE ARE OBTUSE.

Great