Examlet 1B **Foundations of Advanced Math** **2/15/21**

1. The sum of any two consecutive integers is odd.

let's say that n and n+1 are two consecutive integers. Then, the sum of the two would be $n+n+1 = 2n+1$. This results in an integer times 2, plus 1 which is odd by def^n. Therefore, two consecutive integers are odd by def^n. □

Great!

2. Show that if $p \in \mathbb{Z}$ and $3|p^2$ then $3|p$.

Well, $3|p^2$ means $p^2 = 3n$ for some $n \in \mathbb{Z}$.

We know if $p \equiv_3 0$ then $p^2 \equiv_3 0$,
if $p \equiv_3 1$ then $p^2 \equiv_3 1$,
if $p \equiv_3 2$ then $p^2 \equiv_3 1$,

So the only which makes $p^2 = 3n$ for $n \in \mathbb{Z}$ is $p \equiv_3 0$, which is the same as $3|p$. □

3. Determine whether the statements $Q \Rightarrow P$ and $\neg P \vee Q$ are logically equivalent.

Proof:

We construct the following truth table

P	Q	* $Q \Rightarrow P$	$\neg P$	* $\neg P \vee Q$
T	T	T	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Since the statements $Q \Rightarrow P$ and $\neg P \vee Q$ do not have the same truth values under all circumstances, $Q \Rightarrow P$ & $\neg P \vee Q$ are not logically equivalent. $\square$

Great

4. $\sqrt{5}$ is irrational.

Well, s'pose it's rational, so $\exists\, a, b \in \mathbb{Z}$ for which

$$\sqrt{5} = \frac{a}{b}$$

and pick them to have no common factors (reduce if needed).

Then squaring gives

$$5 = \frac{a^2}{b^2}$$

Or $\quad 5b^2 = a^2.$

Then since 5 divides $5b^2$, we must also have $5 \mid a^2$, and it follows from that that $5 \mid a$, so $a = 5n$ for some $n \in \mathbb{Z}$.

Then substituting for a gives

$$5b^2 = (5n)^2$$

or

$$5b^2 = 25n^2$$

or

$$b^2 = 5n^2.$$

But that means 5 divides b^2, hence 5 divides b, contradicting the fact that a and b had no common factors. $\square$

5. For any $n \in \mathbb{Z}^+$,

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$ ← By induction

Prove Base case where $n=1$

$$\sum_{i=1}^{1} (1) = \frac{1(1+1)}{2} = \frac{(1)(2)}{2} = 1$$

The statement holds true for $n=1$. Assume this is true for ~~all~~ sort of. $n=k$, such that:

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

Now, it also needs to hold for $n=k+1$, so we will add $k+1$ to both sides

$$\sum_{i=1}^{k} i + k+1 = \frac{k(k+1)}{2} + k+1$$

[Common denominators

$$\sum_{i=1}^{k+1} i = \frac{k^2+k}{2} + \frac{2k+2}{2}$$

[add together

$$\sum_{i=1}^{k+1} i = \frac{k^2+3k+2}{2}$$

[factor

$$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$$

[manipulate

$$\sum_{i=1}^{k+1} i = \frac{k+1((k+1)+1)}{2}$$

This takes the same form $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ for $n=k+1$ thus true!

By induction, the statement holds true for any $n \in \mathbb{Z}^+$

Okay!