Exam 3a **Foundations of Advanced Math** **3/29/21**

1. The product of an odd function with an even function is odd.

Proof:

Let $f: A \to B$ & $g: B \to C$ be an odd function & an even function respectively so that $f(-x) = -f(x)$ and $g(-x) = g(x)$. Then, we define a new function $(f \cdot g)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot g(x)$ [by our previously stated fact], which can be rewritten as $-(f \cdot g)(x)$, which is an odd function. Hence, the product of an odd function with an even function is odd by definition. □

Great

2. If $f : A \to B$ and $g : B \to C$ are surjective functions, then $g \circ f$ is surjective.

Proof: We have $f: A \to B$ & $g: B \to C$ as two surjective functions

Take $c \in C$ so that $\exists\, b \in B$ for $g(b) = c$.

Now, take the same $b \in B$ so that $\exists\, a \in A$ for $f(a) = b$.

Then,

$$g \circ f(a) = g(f(a)) = g(b) = c$$

Hence $g \circ f$ is surjective by definition.

Good

3. Let $f : A \to B$ be an invertible function. Then f is bijective.

f has an inverse, call it g.

So $\forall b \in B$, $f(g(b)) = b$. Since $g(b) \in A$,

$\forall b \in B, \exists a \in A, f(a) = b$ means f is surjective.

If $f(a_1) = f(a_2)$, then $g(f(a_1)) = g(f(a_2))$ by def of functions. So $g(f(a_1)) = a_1$ and $g(f(a_2)) = a_2$, by def of inverse and so $a_1 = a_2$. therefore, f is injective because $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$.

So f is bijective.

Great

4. (a) A set A is equipollent to itself.

Well, $f(a)=a$ is a bijective function from A to A

So by def, A is equipollent to itself.

Good

(b) If A is equipollent to B, then B is equipollent to A.

If A is equipollent to B there exists a bijective function $f: A \to B$. We know that a bijective function is invertible and its inverse is also a bijection, define this as $g: B \to A$. Since a bijective function $g: B \to A$ exists then if A is equipollent to B, B is equipollent to A also.

Nice

$A = \{n \mid n \in \mathbb{Z} \land n > -6\}$

5. The set $\{n|n \in \mathbb{Z} \land n > -6\}$ is denumerable.

$f: \mathbb{N} \to A,\ f(n) = n - 5$

$g: A \to \mathbb{N},\ g(z) = z + 5$

Proof of inverse

$f(g(z)) = f(z+5) = z+5-5 = z$

$g(f(n)) = g(n-5) = n-5+5 = n$

because g is an inverse for f, f is a bijection. Therefore since a bijection exists from A to $\mathbb{N}$, A is denumerable.

Yep.