Exam 3b **Foundations of Advanced Math** **3/29/21**

1. The composition of an odd function with an even function is even.

Let f be an odd function so that $f(-x) = -f(x)$ $\forall x \in D$
Let g be an even function so that $g(-x) = g(x)$ $\forall x \in D$

Then $f \circ g(-x) = f(g(-x)) = f(g(x)) = f \circ g(x)$
$g \circ f(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = g \circ f(x)$

So, $f \circ g(-x) = f \circ g(x)$ and $g \circ f(-x) = g \circ f(x)$

$\therefore$ the composition of an even & odd function is even.

Excellent!

2. If $f : A \to B$ and $g : B \to C$ are surjective functions, then $g \circ f$ is surjective.

Let $c \in C$. g is surjective such that $\exists b \in B$ $g(b) = c$.
f is surjective such that for this $b \in B$, $\exists a \in A$ where $f(a) = b$

Then $g \circ f(a)$
$= g(f(a))$
$= g(b)$ since $f(a) = b$
$= c$

Since $\forall c \in C$ there exists $\exists a \in A$ that can be found, $g \circ f$ is surjective.

Nice

3. If $f : A \to B$ has an inverse function g, then g has f as an inverse function also.

$f: A \to B$ has the inverse function $g: B \to A$, such that $\forall a \in A$, $g \circ f(a) = a$, and $\forall b \in B$, $f \circ g(b) = b$. For g to have f as an inverse, it has the exact same requirements, so f is the inverse of g. $\square$

Good

4. (a) A set A is equipollent to itself.

Well, $f(a) = a$ is a bijective function from A to A so by def, A is equipollent to itself.

Good

(b) If A is equipollent to B, then B is equipollent to A.

If A is equipullent to B there exists a bijective function $f: A \to B$. We know that a bijective function is invertible and its inverse is also a bijection, define this as $g: B \to A$. Since a bijective function $g: B \to A$ exists then if A is equipollent to B, B is equipollent to A also.

Nice

5. The set of throdd natural numbers is denumerable.

Well, we know a set is only denumerable iff it is equipollent to the set of natural numbers. Take function

$$f(x) = 3x + 1, \quad \forall x \in N.$$

This bijective function will take any natural number and let it become a throdd natural number. Or take inverted function $g(x) = \frac{x-1}{3}$, $\{x \mid x \in N \wedge x \in 3n+1 \ \forall n \in N\}$.

This bijective function takes any throdd natural number and converts it back into a unique natural numbers, so the set of throdd numbers is denumerable.

Good