Examlet 4b **Foundations of Advanced Math** **4/16/21**

1. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $x \sim y \Leftrightarrow |x - y| > 3$.

   (a) Find 3 elements of $\mathbb{Z}$ that are related to 2.

   (b) Find 3 elements of $\mathbb{Z}$ that are not related to 2.

   (c) Determine whether $\sim$ is an equivalence relation.

2. Let $S = \{a, b, c, d, e\}$, and let $\sim = \{(a, a), (b, b), (b, d), (b, e), (c, c), (d, b), (d, d), (d, e), (e, b), (e, d), (e, e)\}$.

   (a) Give the equivalence classes of $\sim$.

   (b) Give the partition associated with $\sim$.

3. Let S be a set and Π a partition of S. Let $\sim$ be a relation on S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$.

   (a) Show $\sim$ is a reflexive relation.

   (b) Show $\sim$ is a symmetric relation.

   (c) Show $\sim$ is a transitive relation.

4. Let S be a set and define a relation on the subsets of S by saying $T \sim U$ iff there exists a bijection from T to U .

   (a) Determine whether $\sim$ is a reflexive relation, and why.

   (b) Determine whether $\sim$ is a symmetric relation, and why.

   (c) Determine whether $\sim$ is a transitive relation, and why.

5. In any graph, the number of vertices of odd degree is even.