Problem Set 5 **Foundations** **Due 3/3/21**

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submit as a pdf on Moodle.

1. For each $x \in \mathbb{R}$, let $Q_x = \{y | x \leq y \leq x + 2\}$. What are:

   (a)
   $$\bigcup_{x \in \{1,2,3\}} Q_x$$

   (b)
   $$\bigcap_{x \in \{1,2,3\}} Q_x$$

   (c)
   $$\bigcup_{x \in \mathbb{N}} Q_x$$

   (d)
   $$\bigcap_{x \in \mathbb{N}} Q_x$$

   (e)
   $$\bigcap_{x \in \mathbb{Z}} Q_x$$

2. For any sets A, B, and C, $(B - A) \subseteq (C - A) \cup (B - C)$.

3. Show that $A \cap \bigcup_{i \in I} B_i = \bigcup_{i \in I} (A \cap B_i)$.

4. Suppose that $a, b, c \in \mathbb{R}$, and $a - c < b - c$. Then $a < b$.

5. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $a - d < b - c$.

6. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$, $c < d$, and $a, c > 0$. Then $a \cdot c < b \cdot d$.

7. Suppose that $a, b \in \mathbb{R}$. If $a < b$ then $a^2 < b^2$.

8. Suppose that $a, b \in \mathbb{R}$, with $a < b$ and $a, b > 0$. Then $\forall n \in \mathbb{N}, a^n \leq b^n$.