Examlet 3 **Advanced Geometry** **4/21/21**

1. a) State the Neutral Area Postulate.

associated with any polygonal region R is a non-negative nomber, call it $\alpha(R)$, that is the area of R s.t. the following are satisfied...

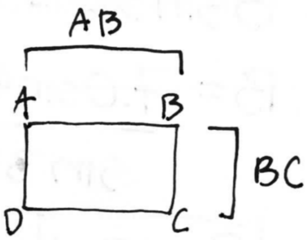
- Congruence: for any two congruent triangles $\Delta ABC \cong \Delta DEF$ their areas are equal s.t. $\alpha(\Delta ABC) = \alpha(\Delta DEF)$
- additivity: for any two non-overlapping polygonal regions, R_1 & R_2, whose union is R, $\alpha(R) = \alpha(R_1) + \alpha(R_2)$

b) State the Euclidean Area Postulate.

for any rectangle, call it $\square ABCD$,

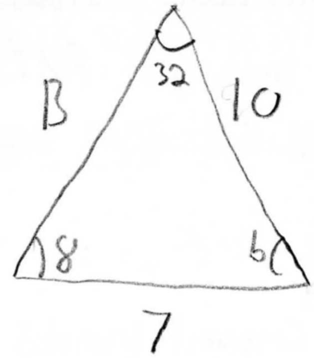
Good

$\alpha(\square ABCD) = AB \cdot BC$



2. A triangle has $\alpha = 32°$, $A = 7.0$, and $C = 10.0$. Solve for the possible remaining measurements, accurate to the nearest tenth.

$$\frac{\sin(\alpha)}{A} = \frac{\sin b}{B} = \frac{\sin 8}{C}$$



Case 1

$$\frac{\sin 32}{7} = \frac{\sin(8)}{10}$$

$$\frac{10 \cdot \sin(32)}{7} = \sin 8$$

$$\sin^{-1}\left(\frac{10 \cdot \sin(32)}{7}\right) = \boxed{8 = 49.2}$$

$b = 180 - 49.2 - 32$

$\boxed{b = 98.8}$

$$\frac{\sin(32)}{7} = \frac{\sin(98.8)}{B}$$

$$\frac{7(\sin(98.8))}{\sin(32)} = \boxed{B = 13.1}$$

Case 2

$8 = 180 - 49.2 = \boxed{130.8 = 8}$

$b = 180 - 32 - 130.8$

$\boxed{b = 17.2}$

$$\frac{\sin(32)}{7} = \frac{\sin(17.2)}{B}$$

$$\frac{7 \cdot \sin(17.2)}{\sin(32)} = \boxed{B = 3.9}$$

Great

10

3. Provide good justifications in the blanks below for the corresponding statements:
Proposition: If $\triangle ABC$ and $\triangle DEF$ are two triangles such that $\triangle ABC \sim \triangle DEF$, then

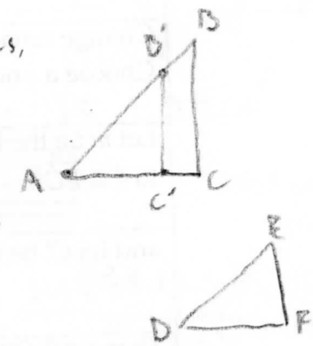
$$\frac{AB}{AC} = \frac{DE}{DF}$$

Statement:	**Reason:**
If $AB = DE$, then $\triangle ABC \cong \triangle DEF$ and the conclusion is evident.	by ASA
So suppose $AB \neq DE$. Either $AB > DE$ or $AB < DE$.	by trichotomy
Change notation, if necessary, so that $AB > DE$. Choose a point B' on $\overline{AB}$ such that $AB' = DE$.	by ruler postulate
Let m be the line through B' such that m is parallel to $\ell = \overleftrightarrow{BC}$	by existence of parallels
and let C' be the point at which m intersects $\overline{AC}$.	by Pasch's Axiom
Then $\angle AB'C' \cong \angle DEF$	by converse of alternate interior angles
Then $\triangle AB'C' \cong \triangle DEF$	by ASA
Let n be the line through A that is parallel to ℓ and m.	by existence of parallels
Then $AB'/AB = AC'/AC$ and so $DE/AB = DF/AC$.	by parallel projection Thrm
$DE/DF = AB/AC$ as desired.	by algebra :)

Great

4. Show that in hyperbolic geometry, two triangles sharing three congruent corresponding angles must be congruent triangles.

Let triangles $\triangle ABC$ and $\triangle DEF$ be similar. Then if they share a side length, by ASA $\triangle ABC \cong \triangle DEF$. So let's assume they share no sides.



Then, because we have 3 boolean outcomes, one of the triangles has two sides which are longer than the other triangle's corresponding sides. Let the longer sides be $\overline{AB}$ and $\overline{AC}$.

Then there exists a point B' on $\overline{AB}$ such that $\overline{AB'} \cong \overline{DE}$ and a point C' on $\overline{AC}$ such that $\overline{AC'} \cong \overline{DF}$.

We know $\square B'BCC'$ is convex, and we also know, by SAS, $\triangle AB'C' \cong \triangle DEF$.

So $\triangle AB'C' \sim \triangle ABC$. Then $\angle AB'C' \cong \angle ABC$ Hm.
and $\angle AC'B' \sim \angle ACB$.

So, because $\angle BB'C'$ and $\angle CC'B'$ are supplements of $\angle B'BC$ and $\angle BCC'$, $\sigma(\square B'BCC') = 360°$

This is a contradiction, so $\triangle ABC \cong \triangle DEF$. □

Good.