Examlet 4 **Foundations of Advanced Math** **4/15/22**

1. (a) State the definition of a reflexive relation.

   (b) Give an example of a relation on the set $\{a, b, c\}$ which is reflexive but not symmetric and not transitive.

2. (a) Suppose that $\sim$ is an equivalence relation on the set $A = \{a, b, c, d, e\}$ and that $[a] = \{a, b, c\}$ and $[d] = \{d, e\}$. Write the partition Π corresponding to $\sim$.

   (b) Suppose that Π is the partition $\{\{1\}, \{2, 4\}, \{3, 5\}\}$ of the set $A = \{1, 2, 3, 4, 5\}$. Find the relation $\sim$ corresponding to Π, expressing it as a set of ordered pairs.

3. Let S be a set and Π a partition of S. Let $\sim$ be a relation on S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$.

   (a) Show $\sim$ is a reflexive relation.

   (b) Show $\sim$ is a symmetric relation.

   (c) Show $\sim$ is a transitive relation.

4. (a) Give all (unlabeled) graphs with $n \leq 4$ vertices.

(b) Give all (unlabeled) trees with $n \leq 4$ vertices.

5. Say that two vertices v_1 and v_2 of a graph G are **propinquous** iff there exists a walk between them that contains exactly one vertex other than v_1 and v_2.

   (a) Is the relation of being propinquous reflexive?

   (b) Is the relation of being propinquous symmetric?

   (c) Is the relation of being propinquous transitive?