Midterm Exam A **Set Theory & Topology** **3/11/22**

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of those you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Don't panic.

1. (a) State the definition of a topology.

   (b) Is the collection of intervals of the form $(a, a+2)$ where $a \in \mathbb{R}$ a topology for $\mathbb{R}$? Why or why not?

2. Show that the composition of continuous functions is continuous

3. Show that the intersection of two closed sets is closed.

4. Suppose that $f : X \to Y$ is a function and $\mathscr{B}$ a basis for Y. Show that f is continuous iff the inverse image of any element of $\mathscr{B}$ is open in X.

5. Let $P = (0, \infty)$. Determine whether each statement is true or false and give a good justification of your answers:

   (a) P is open in $(\mathbb{R}, \mathscr{U})$.

   (b) $P \times P$ is open in $\mathbb{R}^2$ with the product topology.

   (c) $\times\{P : \alpha \in \mathbb{N}\}$ is open in $\times\{\mathbb{R} : \alpha \in \mathbb{N}\}$ with the product topology.

6. Show that the continuous image of a connected set is connected.

7. (a) State the definition of a compact set.

(b) Give an example of a open cover for $(\mathbb{R}, \mathscr{U})$ which has no finite subcover.

□ A. Determine, with justification, if the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x & \text{if } x \geq 1 \\ -2 & \text{if } x < 1 \end{cases}$$

is

(a) $\mathscr{U} - \mathscr{U}$ continuous

(b) $\mathscr{U} - \mathscr{H}$ continuous

(c) $\mathscr{U} - \mathscr{C}$ continuous

(d) $\mathscr{H} - \mathscr{U}$ continuous

(g) $\mathscr{C} - \mathscr{C}$ continuous

☐ B. What is $\mathrm{Cl}((0,1))$ in

(a) $(\mathbb{R}, \mathscr{U})$

(b) $(\mathbb{R}, \mathscr{H})$

(c) $(\mathbb{R}, \mathscr{C})$

(d) $(\mathbb{R}, \mathscr{D})$

(g) $(\mathbb{R}, \mathscr{I})$ (the indiscrete topology)

□ C. Is $(\mathbb{R}, \mathscr{U})$ homeomorphic to $(\mathbb{R}, \mathscr{H})$? Justify your answer well.

□ D. Let $\mathscr{B}$ be a base for a topological space $(X, \mathscr{T})$ and let $A \subseteq X$. Show that the collection $\{B \cap A : B \in \mathscr{B}\}$ is a base for some topology on A.

☐ E. Let $(X, \mathscr{T})$ be a topological space and let $A \subseteq X$. Then A is closed iff $A = \mathrm{Cl}(A)$.

☐ F. $A \times B$ is connected iff A and B are connected.