Problem Set 3 **Set Theory & Topology** **Due 2/7/22**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Ten of these problems will be selected (by Jon) for grading, with each worth 2 points.

1. [Baker Ch 2 R1] The empty set is a closed subset of $\mathbb{R}$ regardless of the topology on $\mathbb{R}$.

2. [Baker Ch 2 R2] Any open interval is an open subset of $\mathbb{R}$ regardless of the topology on $\mathbb{R}$.

3. [Baker Ch 2 R3] Any closed interval is a closed subset of $\mathbb{R}$ regardless of the topology on $\mathbb{R}$.

4. [Baker Ch 2 R4] A half-open interval of the form $[a, b)$ is neither an open set nor a closed set regardless of the topology on $\mathbb{R}$.

5. [Baker Ch 2 R5] If A is a subset of a topological space, then $A \subseteq \mathrm{Cl}(A)$.

6. [Baker Ch 2 R6] If A is a subset of a topological space, then $A' \subseteq A$.

7. [Baker Ch 2 R7] For any closed subset A of a topologoical space, $A' \subseteq A$.

8. [Baker Ch 2 R8] If A is a subset of a topological space, then $\mathrm{Int}(A) \subseteq A$.

9. [Baker Ch 2 R9] For any subset A of a topological space, $\mathrm{Bd}(A) \subseteq A$.

10. [Baker Ch 2 R10] If A is a subset of a topological space, then $\mathrm{Bd}(A) \subseteq \mathrm{Cl}(A)$.

11. [Baker Ch 2 R11] If A is a closed subset of a topological space, then $\mathrm{Bd}(A) \subseteq A$.

12. [Baker Ch 2 R12] If A is a subset of a topological space, then $\mathrm{Int}(A) \subseteq \mathrm{Cl}(A)$.

13. [Baker Ch 2 R13] The point 1 is a limit point of the set $[0, 1)$ regardless of the topology on $\mathbb{R}$.

14. [Baker Ch 2 R14] The point 2 is not a limit point of the set $[0, 1)$ regardless of the topology on $\mathbb{R}$.

15. [Baker Ch 2 R15] For any subset A of a topological space, $\text{Ext}(A) = X - A$.

16. [Baker Ch 2 R16] For any closed subset A of a topological space, $\text{Ext}(A) = X - A$.

17. [Baker Ch 2 R17] The collection $\mathscr{B} = \{\{x\} : x \in \mathbb{R}\}$ is a base for a topology on $\mathbb{R}$.

18. [Baker Ch 2 R18] The collection $\mathscr{B} = \{\{x\} : x \in \mathbb{R}\}$ is a base for the usual topology on $\mathbb{R}$.

19. [Baker Ch 2 R19] In a space $(X, \mathscr{T})$ any collection of open sets whose union equals X and that is closed under finite intersection is a base for $\mathscr{T}$.

20. [Baker Ch 2 R20] There exists a topological space $(X, \mathscr{T})$ such that there is no base for $\mathscr{T}$.

21. [Baker Ch 2 R21] There exists a topological space $(X, \mathscr{T})$ for which there is more than one base for $\mathscr{T}$.