

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Suppose that $f(x) = (\log_5 x)(2^x)$. Find $f'(x)$.

2. Let $f(x) = 7x \sin^{-1}(x)$.

(a) Find $f'(x)$.

(b) Find $f'(0.6)$.

3. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	6	3	5	1	4	2
$g(x)$	2	1	6	3	5	4
$f'(x)$	2	4	1	5	7	8
$g'(x)$	5	9	7	11	2	12

(a) If $h(x) = \tan^{-1}(f(x))$, what is $h'(2)$ and why?

(b) If $h(x) = f(x) \cdot \ln x$, what is $h'(5)$ and why?

(c) If $h(x) = e^{g(x)}$, what is $h'(4)$ and why?

4. Why is the derivative of $\ln x$ equal to $\frac{1}{x}$?

5. A train leaves Boston heading west at noon travelling 24 mi/hr. A second train is heading toward Boston from the north at 18 mi/hr and will arrive at 4pm. How fast is the distance between the trains changing at 2pm?

6. Why is the derivative of $\arctan x$ equal to $\frac{1}{1+x^2}$?

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Dude! I just broke calculus! There was this question where we had to do the derivadid of \ln of $5x$, right? So I did it and when I simplified, like cancelled, I got 1 over x . So obviously that's not possible, because just \ln of x has that for its derivadid, right? So there's no way two different functions have the same derivadid, right?"

Help Biff by explaining as clearly as you can why this isn't really surprising.

8. Carbon dioxide emissions from China have been estimated at 10.30 gigatons in 2016 and 11.30 gigatons in 2020. [World Resources Institute data]

(a) Find a function of the form $f(x) = Ab^x$ for these emissions, where x is in years after 2016.

(b) What does your formula project the carbon dioxide emissions will be in 2030?

(c) What does your formula project the rate of growth of carbon dioxide emissions will be in 2030?

9. Let $F(x) = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x$. Find $F'(x)$.

10. A training program models participants' learning curves with the function

$$P(t) = \frac{2.9e^t}{5.0 + 0.22e^t}$$

where t is the number of training days and $P(t)$ is the participant's expected performance rating.

(a) Find $P^{-1}(x)$

(b) Find the slope of the tangent line to $P^{-1}(x)$ at $x = 13$.

Extra Credit (5 points possible): What do the answers to #10 actually mean?