Exam 4 **Calc 1** **4/14/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Given the following information about a function $f(x)$ which is continuous and differentiable, identify the x-values of all relative maximums, relative minimums, and inflection points of the function.

relative min: $X=-2$
relative max: $X=5$
inflection point: $X=3$

interval	$f'(x)$	$f''(x)$
$(-\infty,-2)$	-	+
$(-2,3)$	+	+
$(3,5)$	+	-
$(5,\infty)$	-	-

when $f''(x)$ goes from pos to neg or vice versa it's an inflection point

max's and mins occur when $f'(x)$ switches from neg to pos (min) or pos to neg (max)

Excellent!

2. Find the interval(s) on which $f(x) = 3x^2 - 6x$ is decreasing.

$f(x) = 3x^2 - 6x$

$f'(x) = 6x - 6$

$0 = 6(x-1)$

$x = 1$

− +

1

test points

$f'(-1) = -$

$f'(2) = +$

Great

$f(x)$ is decreasing at $(-\infty, 1)$

3. Find the interval(s) where $g(x) = 14 + x^2 - x^3$ is concave up.

$g'(x) = 2x - 3x^2$

$g''(x) = 2 - 6x$

$2 - 6x = 0$

$\frac{6x}{6} = \frac{2}{6}$

$x = \frac{1}{3}$

$\left(-\infty, \frac{1}{3}\right) \quad \left(\frac{1}{3}, \infty\right)$

Guess : $-1 \quad 1$

$g''(-1) = 8$

$g''(1) = -4$

$(-\infty, \frac{1}{3})$	+	Concave up
$(\frac{1}{3}, \infty$	−	Concave down

Great!

$\therefore$ The interval for concave up is $\left(-\infty, \frac{1}{3}\right)$

4. Find f if $f'(x) = 8x^3 + 14x + 10$ and $f(1) = -4$.

$f(x) = \frac{8}{4}x^4 + \frac{14}{2}x^2 + 10x + C$

$f(x) = 2x^4 + 7x^2 + 10x + C$

$f(1) = -4 = 2x^4 + 7x^2 + 10x + C$

$= 2(1)^4 + 7(1)^2 + 10(1) + C$

$-4 = 2 + 7 + 10 + C$

$-23 = C.$

$f(x) = 2x^4 + 7x^2 + 10x - 23$

Good

5. Let $f(x) = x^2 - 6x + 3$. Find the absolute maximum and absolute minimum values (heights) of f on $[1, 5]$.

$$f(x) = x^2 - 6x + 3$$

$$f'(x) = 2x - 6$$

$$0 = 2(x-3)$$

$$x = 3$$

maximum : -2

Minimum : -6

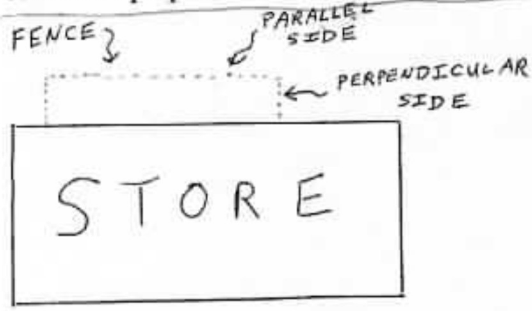
$$f(1) = 1^2 - 6(1) + 3 = -2$$

$$f(3) = 3^2 - 6(3) + 3 = -6$$

$$f(5) = 5^2 - 6(5) + 3 = -2$$

Good

6. [WW] The owner of a garden supply store wants to construct a fence to enclose a rectangular outdoor storage area adjacent to the store, using part of the side of the store (which is 270 feet long) for part of one of the sides. There are 450 feet of fencing available to complete the job. Find the length of the sides parallel to the store and perpendicular that will maximize the total area of the outdoor enclosure.



Set perpendicular side is x

Parallel side is y

$y \le 270$

$450 - 2x \le 270$

$-2x \le -180$

$2x \geq 180$

$x \geq 90$

$2x + y = 450$

$y = 450 - 2x$

$S = xy$

$= x(450 - 2x)$

$= -2x^2 + 450x$

$S' = -4x + 450 = 0$

$-4x = -450$

$x = 112.5$

So $y = 450 - 2 \times 112.5 = 225$.

So the parallel side is 225 feet

perpendicular side is 112.5 feet

Excellent!

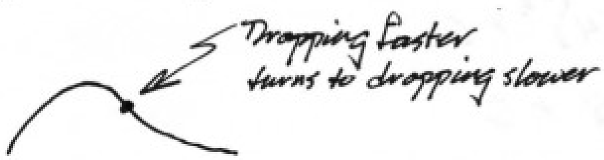
7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Man, this calculus stuff is tough. I was doing pretty good with the critical stuff and maxes and stuff, but then this infliction point thing is just totally confusing. Is it just the same as the critical stuff or what?"

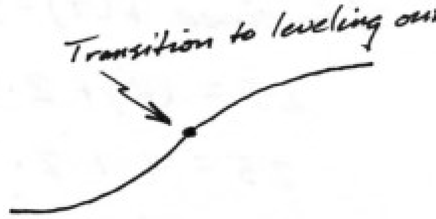
Explain clearly to Biff what an inflection point is, how to find one, and why they might matter.

So Biff, an inflection point is a place where a function goes from concave up to concave down, or vice versa. You normally find them by setting the second derivative equal to zero, since for it to change signs it has to pass zero.

What it means depends on context, but generally it's telling you something important about the shape of the graph. It could be about an economic downturn starting to ease up, or an epidemic starting to spread slower instead of increasingly faster. In a medical context it's sometimes understood as about peak sensitivity to some treatment. They can be very important to predicting what comes next, or how soon to expect something.

I hope this helps!





8. Find the x-values of *all* relative maximums and minimums of $y = \frac{x}{2} + \cos x$.

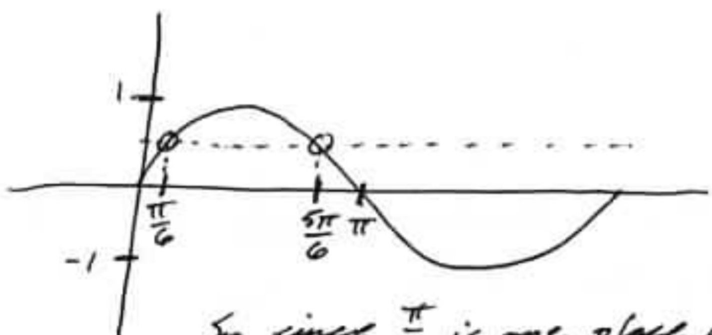
I. Take the derivative: $y' = \frac{1}{2} + -\sin x$

II. Set equal to zero: $0 = \frac{1}{2} - \sin x$

$$\sin x = \frac{1}{2}$$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = \frac{\pi}{6}$$



So since $\frac{\pi}{6}$ is one place we get a height of $\frac{1}{2}$ on the sine graph, $\pi - \frac{\pi}{6}$ is the next time, an equal distance before finishing that first hump. But then sine repeats every 2π, so things like $\frac{\pi}{6} + 2\pi$ and $\frac{\pi}{6} + 4\pi$ get that same height.

Therefore

$$\frac{\pi}{6} + 2\pi k \text{ and } \frac{5\pi}{6} + 2\pi k$$

for any integer k, will be a relative max or min of this graph.

9. Use Newton's method to find the second and third approximation of a root of

$$x^3 - 4x - 3 = 0$$

starting with $x_1 = 2$ as the initial approximation.

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

$f(x) = x^3 - 4x - 3$

$f'(x) = 3x^2 - 4$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{-3}{8}$$

$$= \frac{19}{8} \approx 2.375$$

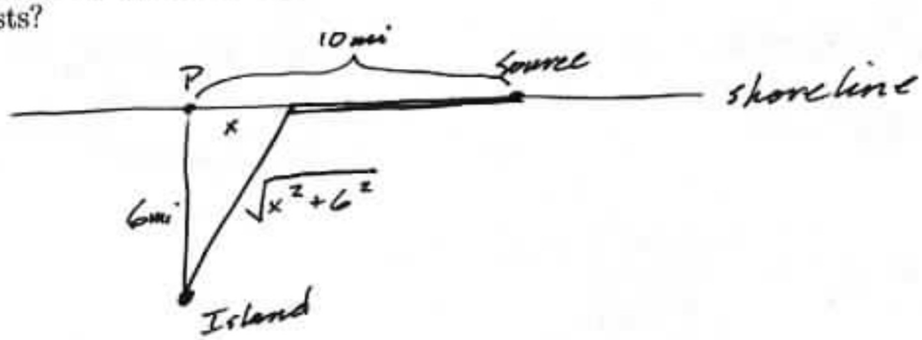
$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= \frac{19}{8} - \frac{f(\frac{19}{8})}{f'(\frac{19}{8})}$$

$$\approx \frac{19}{8} - \frac{0.89648}{12.921875}$$

$$\approx 2.305622733$$

10. [WW] A small resort is situated on an island that lies exactly 6 miles from P, the nearest point to the island along a perfectly straight shoreline. 10 miles down the shoreline from P is the closest source of fresh water. If it costs 1.7 times as much money to lay pipe in the water as it does on land, how far down the shoreline from P should the pipe from the island reach land in order to minimize the total construction costs?



$$\text{Cost} = 10 - x + 1.7 \cdot \sqrt{x^2 + 6^2}$$

$$C'(x) = -1 + 1.7 \cdot \tfrac{1}{2}(x^2+6^2)^{-1/2} \cdot 2x$$

$$0 = -1 + \frac{1.7x}{\sqrt{x^2+36}}$$

$$1 = \frac{1.7x}{\sqrt{x^2+36}}$$

$$\sqrt{x^2+36} = 1.7x$$

$$x^2 + 36 = 2.89x^2$$

$$36 = 1.89x^2$$

$$\Rightarrow x^2 = \frac{36}{1.89}$$

$$x^2 \approx 19.04761905$$

$$x \approx 4.364357805$$