Exam 1 **Calculus 2** **2/1/23**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Evaluate $\int xe^x \, dx$

$u = x \quad v = e^x \qquad u \cdot v - \int u' \cdot v \, du$

$u' = 1 \quad v' = e^x$

$xe^x - \int 1 \cdot e^x \, dx$

$xe^x - e^x + C$

Good

2. Evaluate $\int \sin^3\theta \cos\theta \, d\theta$

$\int \sin^3\theta \cos\theta \, d\theta$

Let $u = \sin\theta$

$\frac{du}{dx} = \cos\theta$

$du = \cos\theta \, d\theta$

$= \int u^3 \, du$

Great!

$= \frac{1}{4}u^4 + C$

$= \frac{1}{4}\sin^4\theta + C$

3. Write the appropriate form for a partial fractions decomposition of the function

$$\frac{2(x^4+1)}{(x-2)(x-1)^2(x^2+2)^2}$$

$$\frac{2(x^4+1)}{(x-2)(x-1)^2(x^2+2)^2} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2} + \frac{Dx+E}{(x^2+2)} + \frac{Fx+G}{(x^2+2)^2}$$

Good

4. Evaluate $\int_1^{e^3} \frac{dx}{x(1+\ln x)}$

$$\int_1^{e^3} \frac{1}{x(1+\ln x)}\,dx$$

Let $u = 1 + \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{dx}{x}$

$dx = x\,du$

$1 + \ln(1) = 1 + 0 = 1$

$1 + \ln(e^3) = 1 + 3 = 4$

$$\int_1^4 \frac{1}{\cancel{x}u} \cdot \cancel{x}\,du$$

$$= \int_1^4 \frac{1}{u}\,du$$

$$= \ln|u|\Big|_1^4 = \ln|4| - \ln|1| = \ln(4)$$

5. Evaluate $\int_2^{\infty} e^{-5p}\,dp$

$\rightarrow \infty$ means limits

$$\int_2^{\infty} e^{-5p}\,dp = \lim_{b\to\infty} \int_2^{b} e^{-5p}\,dp$$

Let $u = -5p$

$\frac{du}{dp} = -5$

$du = -5\,dp$

$dp = \frac{-du}{5}$

$$= \lim_{b\to\infty} \left(-\tfrac{1}{5}\right)\int_2^{b} e^{u}\,du$$

$$= \lim_{b\to\infty} \left(-\tfrac{1}{5}\right) e^{u} \Big|_2^{b}$$

$$= \lim_{b\to\infty} \left(-\tfrac{1}{5}\right) e^{-5p} \Big|_2^{b} = \lim_{b\to\infty} \left(-\tfrac{1}{5} e^{-5b} + \tfrac{1}{5} e^{-10}\right)$$

$$= \lim_{b\to\infty} \left(-\underbrace{\frac{1}{5e^{5b}}}_{\text{approaches } 0} + \frac{1}{5e^{10}}\right)$$

Excellent!

$$= \frac{1}{5}e^{10}$$

6. Evaluate $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

$u + x^2$

$\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

$u = 1 - x^2$ $\quad$ $x^2 = 1 - u$

$\frac{du}{dx} = -2x$

$dx = \frac{du}{-2x}$

$\int \frac{x^3}{\sqrt{u}}\,dx$

$\int \frac{x^{\cancel{3}}}{\sqrt{u}}\,\frac{du}{-2\cancel{x}}$

$-\frac{1}{2}\int \frac{x^2}{\sqrt{u}}\,du$

$-\frac{1}{2}\int \frac{1-u}{\sqrt{u}}$

$-\frac{1}{2}\int u^{-1/2}(1-u)$

$-\frac{1}{2}\int u^{-1/2} - u^{1/2}$

$-\frac{1}{2}\left(2u^{1/2} - \frac{2}{3}u^{3/2}\right)$

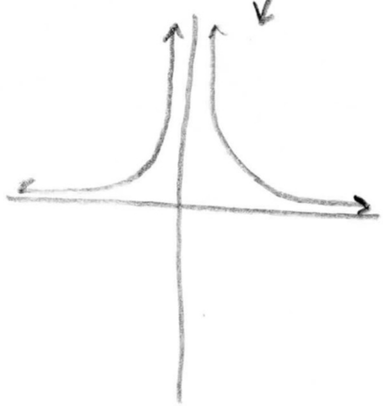
$-\frac{1}{2}\left(2\sqrt{1-x^2} - \frac{2}{3}\sqrt{(1-x^2)^3}\right. + C$

Excellent!

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, Calc 2 is the hardest class evvarr! We had this one problem on our first test, like integrate 1 over x squared from -2 to 5, right? And I did the antideriva-thingie, then plugged in 5 and -2 just like usual, right? And they gave me *no* credit! Not even partial anything! I think they just want everyone to drop so they can have more time for their research, right?"

   Help Bunny out by explaining what might have led to the harsh grading (beyond selfish faculty).

In doing this problem, it would be helpful to look at the graph of $\frac{1}{x^2}$. The graph looks something like this. There is an asymptote



at $x=0$, so the integral on it at the given range would be infinite. You would have to split up the integral into two parts that exclude $x=0$.

Good

8. Derive Line 20 from the Table of Integrals,

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

$$\int \frac{1}{u^2-a^2}$$

$$\frac{1}{u^2-a^2} = \frac{A}{u-a} + \frac{B}{u+a} \quad (u-a)(u+a)$$

$$1 = A(\overset{a}{u}+a) + B(u-a) \qquad u=a$$

$$\frac{Aa^2}{2a} + 0$$

$$\underline{\frac{1}{2a} = A}$$

$$u=-a$$

$$1 = A(\overset{-a}{u}+a) + B(\overset{-a}{u}-a)$$

$$\underline{B = \frac{1}{-2a}} \qquad 0 + B(-2a)$$

$$1 = \frac{\frac{1}{2a}}{u-a} + \frac{-\frac{1}{2a}}{u+a}$$

$$\frac{1}{2a} \ln|u-a| - \frac{1}{2a} \ln|u+a|$$

$$\underline{\frac{1}{2a} \ln\left|\frac{u-a}{u+a}\right| + C}$$

Great

$\sin^2\theta + \cos^2\theta = 1$
$\tan^2\theta + 1 = \sec^2\theta$

9. Derive Line 17 from the Table of Integrals,

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \int \frac{a\sec^2\theta}{a^2+a^2\tan^2\theta}\,d\theta$$

$$= \frac{1}{a}\int \frac{\sec^2\theta}{1+\tan^2\theta}\,d\theta$$

$$= \frac{1}{a}\int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{a}\int 1\,d\theta$$

$$= \frac{1}{a}\theta + C$$

$$= \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

Let $u = a\tan\theta$

$\frac{du}{d\theta} = a\sec^2\theta$

$du = a\sec^2\theta\,d\theta$

$\frac{u}{a} = \tan\theta$

$\tan^{-1}\frac{u}{a} = \theta$

10. Evaluate $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = \int \frac{x}{\sqrt{-1(x^2+2x+1)+3+1}}\,dx$

$$= \int \frac{x}{\sqrt{-1(x+1)^2+4}}\,dx$$

$$= \int \frac{u-1}{\sqrt{4-u^2}}\,du$$

Let $u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

$$= \int \frac{u}{\sqrt{4-u^2}}\,du - \int \frac{1}{\sqrt{4-u^2}}\,du$$

$$= \int \frac{\cancel{u}}{\sqrt{v}} \cdot \frac{dv}{-2\cancel{u}} - \sin^{-1}\frac{u}{2}$$

Let $v = 4-u^2$

$\frac{dv}{du} = -2u$

$\frac{dv}{-2u} = du$

$$= -\frac{1}{2}\int v^{-1/2}\,dv - \sin^{-1}\frac{u}{2}$$

$$= -\frac{1}{2}\cdot 2\cdot v^{1/2} - \sin^{-1}\frac{u}{2} + C$$

$$= -\sqrt{4-u^2} - \sin^{-1}\frac{u}{2} + C$$

$$= -\sqrt{3-2x-x^2} - \sin^{-1}\left(\frac{x+1}{2}\right) + C$$