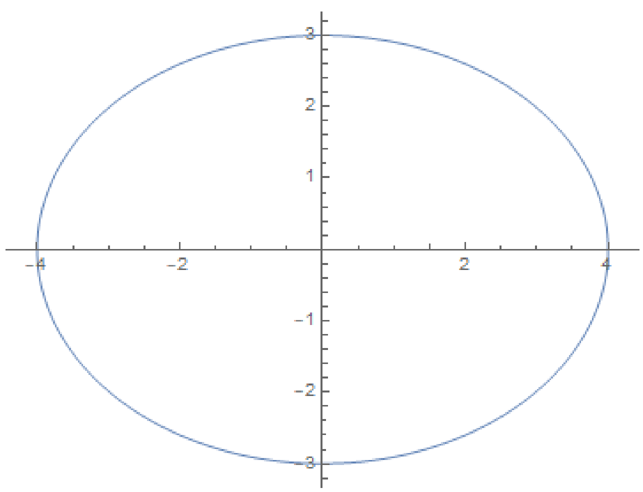
Exam 4 **Calculus 2** **4/21/22**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Convert the point with rectangular coordinates $(0, -5)$ to polar coordinates (r, θ).

2. Find an equation for the ellipse shown:



3. Find the slope of the tangent line to the polar curve $r = \sin(4\theta)$ at $\theta = \frac{\pi}{8}$.

4. Find the slope of the tangent line to the lemniscate with parametric equations $x(t) = 2\sin\frac{t}{2}$ and $y(t) = \sin t$ at the point $\left(1, \frac{\sqrt{3}}{2}\right)$.

5. Write an integral for the area of the region inside the inner loop of $r = 1 + 2\cos(\theta)$.

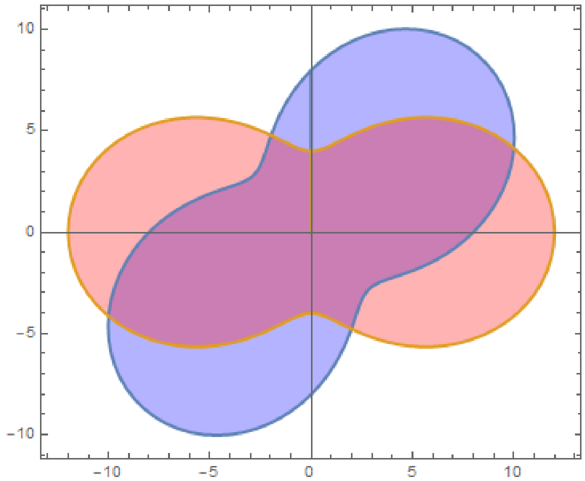
6. Identify the graph of $x^2 - 4y^2 - 6x + 8y + 1 = 0$ as a parabola, hyperbola, or ellipse, give coordinates of its vertices, and sketch a decent graph including any asymptotes.

7. Bunny is a Calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod! Polar thingies are so pretty! But I'm totally geting stuck on the limits for integrating them, you know? I mean, 0 to 1 is always what I guess first, right, but that never seems to be right with these, so I guess 0 to 2 pi is like, normal or something? But sometimes it was just to pi or to 4 pi. How are you supposed to guess which?"

   Help Bunny out by explaining how to determine the appropriate limits of integration for curves in polar coordinates.

8. Write an integral for the length of the lemniscate with parametric equations $x(t) = 2\sin\frac{t}{2}$ and $y(t) = \sin t$.

9. Set up an integral for the area of the blue region in the plot below of $r = 8 + 4\cos 2\theta$ and $r = 8 + 4\sin 2\theta$, so the portion outside the horizontally oriented curve but inside the diagonally oriented curve.



10. Consider the curve with parametric equations $x(t) = 48 - 4t^2$, $y(t) = t^5 - 10t^3 + 9t$. Find the area of the entire region bounded by this curve.

Extra Credit [5 points possible]:

For the curve with parametric equations $x(t) = 48 - 4t^2$, $y(t) = t^5 - 10t^3 + 9t$ from #10, find an equation for a line that is tangent to the curve in exactly two places.