Examlet 4 **Foundations of Advanced Math** **4/14/23**

1. Consider the relation $\sim$ on $\mathbb{Z}$ defined by $a \sim b \Leftrightarrow 3|(a-b)$.

   (a) Determine whether and why $\sim$ is reflexive.

   (b) Determine whether and why $\sim$ is symmetric.

   (c) Determine whether and why $\sim$ is transitive.

2. Let $S = \{a, b, c, d, e\}$, and let $\sim = \{(a, a), (a, e), (b, b), (b, c), (c, b), (c, c), (d, d), (e, a), (e, e)\}$.

   (a) Give the equivalence classes of $\sim$.

   (b) Give the partition associated with $\sim$.

3. Let S be a set and Π a partition of S. Let $\sim$ be a relation on S defined by $a \sim b \Leftrightarrow \exists P \in \Pi$ for which $a, b \in P$.

   (a) Show $\sim$ is a reflexive relation.

   (b) Show $\sim$ is a symmetric relation.

   (c) Show $\sim$ is a transitive relation.

4. Biff is a student at Enormous State University who has inexplicably found himself in a transition-to-proof course. Biff says "So we had to say if these things were, like, refluxive and symmetry and transitive, right? And one of them was if $a \sim b \Leftrightarrow a + b \equiv_5 0$, and I said it was refluxive because $5 \sim 5$, right? But they gave me no credit at all and then made fun of my answer in class the next day as an example of how not to ever do anything. I think I'm gonna drop."

   Explain clearly to Biff what's wrong with his deduction.

5. For two vertices v_1 and v_2 of a graph G we say $v_1 \sim v_2 \Leftrightarrow \exists$ a walk of even length from v_1 to v_2.

   (a) Determine whether and why $\sim$ is reflexive.

   (b) Determine whether and why $\sim$ is symmetric.

   (c) Determine whether and why $\sim$ is transitive.