

Four of these problems will be graded (our choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but you must write up your own final submission without reference to any sources other than the textbook and instructor.

1. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are even, then  $f - g$  is even.
2. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are odd functions with  $g(x) \neq 0 \forall x \in \mathbb{R}$ , then  $f/g$  is an odd function.
3. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are increasing, then  $f \cdot g$  is increasing.
4. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  are strictly decreasing, then  $f + g$  is strictly decreasing.
5. If  $f : \mathbb{R} \rightarrow \mathbb{R}$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  have negative derivatives  $\forall x \in \mathbb{R}$ , then  $f + g$  is decreasing.
6. Let  $f$  and  $g$  be bounded functions, both with domain  $D$ . Then  $f + g$  is a bounded function.
7. Let  $f$  and  $g$  be bounded functions, both with domain  $D$ . Then  $f - g$  is a bounded function.
8. Let  $f$  and  $g$  be bounded functions, both with domain  $D$ . Then  $f \cdot g$  is a bounded function.
9. Let  $f$  and  $g$  be bounded functions, both with domain  $D$ . Then  $f/g$  is a bounded function.
10. Let  $n \in \mathbb{N}$ , and  $f_i$  be an odd function with domain  $D$  for each  $i \in \{x \in \mathbb{N} \mid 1 \leq x \leq n\}$ . Then  $\sum_{i=1}^n f_i$  is an odd function.
11. Critique the following proof of the proposition “The product of two odd functions, both with domain  $D$ , is odd”:  
Let  $f$  and  $g$  be two odd functions, both with domain  $D$ , and consider their product  $f \cdot g$ . Then,

$$(f \cdot g)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot (-g(x)) = -(f(x) \cdot g(x)) = -(f \cdot g)(x).$$

Therefore,  $f \cdot g$  is odd.