Exam 2b **Calculus 2** **3/1/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Set up an integral for the area of the region bounded between $y = x^2$ and $y = 3x$.

2. Set up an integral for the volume of the solid obtained when the region from #1 is rotated around the x-axis.

3. If the work required to stretch a spring 1 ft beyond its natural length is 20 ft-lb, how much work (in ft-lb) is needed to stretch it 6 in. beyond its natural length?

4. Set up an integral for the length of the portion of the curve $y = \sqrt{x}$ from $(0, 0)$ to $(9, 3)$.

5. Write integrals for the present and future values of an income stream of 3000 dollars a year, for a period of 5 years, if the continuous interest rate is 4 percent.

6. Find the x-coordinate of the center of mass of the region lying underneath the graph of the function $f(x) = \sqrt{x}$ over the interval $[0, 16]$.

7. Star is a calculus student at Enormous State University, and they're having some trouble. Star says "Geez! Calc 2 is so different from Calc 1! It used to be there was, like, just one right way to do things, right? But now I did this work problem as $\int_5^{10} 250\pi(10-x)\,dx$ but the professor was saying to do it $\int_0^5 250\pi x\,dx$. The thing is, they both worked out to the same answer! Is it just a coincidence?"

   Help Star out. Explain to them as clearly as possible why both ways work for the same problem, or why you know it's just a coincidence.

8. A gas station stores its gasoline in a tank under the ground. The tank is a cylinder lying horizontally on its side. (In other words, the tank is not standing vertically on one of its flat ends.) If the radius of the cylinder is 0.5 meter, its length is 2 meters, and its top is 1 meter under the ground, set up an integral for the total amount of work needed to pump the gasoline out of the tank to ground level. (The density of gasoline is 673 kilograms per cubic meter; use $g = 9.8m/s^2$).

9. Consider the entire region bounded between $y = x^3$ and $y = 3x$. If this region is revolved around the axis $x = 2$, set up an integral or integrals for the volume of the resulting solid.

10. Consider a sphere with radius r. Treating it as a solid of revolution obtained from rotating half of a circle centered at the origin around one of the coordinate axes, show that the surface area of the sphere is $4\pi r^2$.

Extra Credit [5 points possible]: There is a line through the origin that divides the region bounded by the parabola $y = x - x^2$ and the x-axis into two regions with equal area. What is the slope of that line?