Exam 3b **Calculus 2** **3/29/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Determine the exact sum of the geometric series $4 - 2 + 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

2. Find the first 3 partial sums of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)!}$

3. Find the 5th degree MacLaurin polynomial for $f(x) = e^x$.

4. Determine whether the series $\sum_{n=2}^{\infty} \frac{3}{n \ln n}$ converges or diverges.

5. Determine whether the series $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{\sqrt{n}}$ converges or diverges.

6. Write the first 3 non-zero terms of the Taylor Series for $f(x) = \sin x$ centered at $x = \frac{\pi}{2}$.

7. Biff is a Calculus student at Enormous State University, and he's having some trouble. Biff says "Dude, this series stuff is crazy. So we were supposed to say if $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$ converges or diverges, right? So I said it converges by comparison to $\sum_{n=2}^{\infty} \frac{1}{n^2}$, which was totally one of the choices on the multiple choice, right? But they said it was wrong! They're like practically the same thing, so what could be wrong about it?"

   Help Biff out by explaining the validity of his conclusion. You do not need to say whether the series converges or not; you're just commenting on the proposed reasoning.

8. Use a Maclaurin polynomial with at least 4 terms to approximate $e^{0.1}$.

9. Use a Maclaurin polynomial of at least 8^{th} degree to approximate $\int_0^1 \cos(x^2)\,dx$

10. For which values of x does $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^n$ converge?

Extra Credit [5 points possible]: Find the exact value of $1 - \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \cdots$, where the sign pattern repeats three positive terms then one negative?