Examlet 2b **Foundations of Advanced Math** **2/23/24**

1. Write each of the following in as simple a way as possible:

   (a) $(5, 10) - [6, 12]$

   $(5, 6)$

   (b) $[5, 10] - [6, 12]$

   $[5, 6)$

Circle T or F for each of the following statements:

(c) $0 \subseteq \{0, 1, 2\}$ T F

0 is not a set,
it is an element

(d) $0 \in \{0, 1, 2\}$ T F

(e) $\{0\} \in \{0, 1, 2\}$ T F

0 is an element
$\{0\}$ is a subset

Great

$\{0\}$ is not an element

$$\left[-\frac{1}{n}, \frac{1}{n}\right] = A_n$$

2. (a) What is $\bigcap_{n\in\{1,2,3\}} \left[-\frac{1}{n}, \frac{1}{n}\right]$?

$A_1 = [-1, 1]$ ← they all contain $\left[-\frac{1}{3}, \frac{1}{3}\right]$

$A_2 = \left[-\frac{1}{2}, \frac{1}{2}\right]$

$A_3 = \left[-\frac{1}{3}, \frac{1}{3}\right]$

$$\bigcap_{n\in\{1,2,3\}} A_n = \left[-\frac{1}{3}, \frac{1}{3}\right]$$

Excellent!

(b) What is $\bigcup_{n\in\{1,2,3\}} \left[-\frac{1}{n}, \frac{1}{n}\right]$?

$$\bigcup_{n\in\{1,2,3\}} A_n = [-1, 1]$$

* what contains everything in the set

(c) What is $\bigcap_{n\in\mathbb{Z}^+} \left[-\frac{1}{n}, \frac{1}{n}\right]$?

$$\bigcap_{n\in\mathbb{Z}^+} A_n = \{0\}$$

x (d) What is $\bigcup_{n\in\mathbb{Z}^+} \left[-\frac{1}{n}, \frac{1}{n}\right]$?

$n = 1$

$$\bigcup_{n\in\mathbb{Z}^+} A_n = [-1, 1]$$

3. $\left(\bigcap_{i\in I} B_i\right)' = \bigcup_{i\in I} B_i'$.

Let $x \in \left(\bigcap_{i\in I} B_i\right)'$ $\Leftrightarrow$ $\neg\left(x \in \bigcap_{i\in I} B_i\right)$ $\Leftrightarrow$ $\neg(\forall i \in I, x \in B_i)$

$\Leftrightarrow$ $(\exists i \in I, x \notin B_i)$ $\Leftrightarrow$ $(\exists i \in I, x \in B_i')$ $\Leftrightarrow$ $x \in \bigcup_{i\in I} B_i'$,

so $\left(\bigcap_{i\in I} B_i\right)' \subseteq \bigcup_{i\in I} B_i'$. since each statement is logically equivalent and reversible, $\bigcup_{i\in I} B_i' \subseteq \left(\bigcap_{i\in I} B_i\right)'$, and therefore $\left(\bigcap_{i\in I} B_i\right)' = \bigcup_{i\in I} B_i'$. $\square$

Great

4. $\forall r \in \mathbb{R}, r > 1 \Rightarrow r > \frac{1}{r} > \frac{1}{r^2}$

multiply $\frac{1}{r}$ on both sides. $\frac{1}{r}$ must >0,

proof From CMP def^n we can get, if we ✓ then the multiply can be used. Suppose $\frac{1}{r} < 0$ when $r > 1$. then we let both sides multiplied by r. $\frac{1}{r} \cdot r < 0 \cdot r \Rightarrow 1 < 0$ this is not correct, so we use contradiction to prove $\frac{1}{r} > 0$. Good!

Then we can finally use CMP. both side multiplied by $\frac{1}{r}$,

then we get $r \cdot \frac{1}{r} > 1 \cdot \frac{1}{r} \Rightarrow 1 > \frac{1}{r}$

Multiplied again $1 \cdot \frac{1}{r} > \frac{1}{r} \cdot \frac{1}{r} \Rightarrow \frac{1}{r} > \frac{1}{r^2}$

Therefore, ve get $r > 1$, $1 > \frac{1}{r}$, $\frac{1}{r} > \frac{1}{r^2}$, it can be rewritten as $r > 1 > \frac{1}{r} > \frac{1}{r^2}$, which can be used TIP's def^n

Hince, $r > \frac{1}{r} > \frac{1}{r^2}$ is correct. □

Well done!

5. $\forall x, y \in \mathbb{R}, |x| \leq y \Rightarrow -y \leq x \leq y$.

Using Lemma 1 we can expand x to $-|x| \leq x \leq |x|$

where we know $|x| \leq y$. Using the definition of an absolute value, we know $|x| \geq 0$, therefore $y \geq 0$.

If $x \geq 0$.

Lemma 1 tells us $-|y| \leq x \leq |x| \leq y$.

$x \leq y \iff x+(-y) \leq y+(-y) \iff -y+x+(-x) \leq y+(-y)+(-x) \iff -y \leq -x$ via comp. ad. princ.

and since $|x| = x$ by def. when $x \geq 0$ $-|x| = -x$ and $-y \leq -x$

$\therefore -y \leq -|x| \leq x \leq |x| \leq y$

If $x < 0. \iff x+(-x) < 0+(-x) \iff 0 < -x$ via comp ad. princ.

Lemma1 tells us $-|x| \leq x \leq |x| \leq y$. By def. if $x<0$, $|x| = -x$

So $|x| \leq y \iff -x \leq y \iff -x+(-y)+x \leq y+x+(-y) \iff -y \leq x$ via comp. ad. prince

Now $-|x| = -(-x)$ by def $\iff x$ and we know $-y \leq x$

$\therefore -y \leq -|x| \leq x \leq |x| \leq y$.

So, for all cases $|x| \leq y \Rightarrow -y \leq x \leq y$.

Carl