Problem Set 1   **Foundations**   **Due 1/22/24**

Four of these problems will be graded (my choice, not yours!), with each problem worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but you must write up your own final submission without reference to any sources other than the textbook and instructor.

1. The difference of any two odd integers is even.

2. The cube of an odd integer is odd.

3. If $p, q, r \in \mathbb{Z}$ with p, q, and r odd, then $p + q + r$ is odd.

4. If $p, q, r \in \mathbb{Z}$ with $p + q + r$ odd, then p, q, and r are odd.

5. There is no integer which is both even and odd.

6. The sum of two throddodd integers is throdd.

7. The product of two throddodd integers is throdd.

8. If $n^3 \in \mathbb{Z}$ is throdd, then n is throdd.

9. The sum of any three consecutive integers is threven.

10. Critique the following proof of the proposition:

    Proposition: The product of an even integer and an odd integer is even.

    Proof: Well, let m be an even integer and let n be an odd integer, so $m = 2a$ and $n = 2a + 1$, where $a \in \mathbb{Z}$. We can see that $a(2a + 1)$ is an integer by the closure of the integers under addition and multiplication. Then $mn = 2a(2a + 1)$, and so mn is 2 times an integer and thus even by definition. $\square$