Problem Set 1 **Set Theory & Topology** **Due 2/5/24**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Four of these problems will be selected (by Jon) for grading, with each worth 5 points.

1. [Baker 2.1.5] Prove that if F is any finite subset of $\mathbb{R}$. then $\mathbb{R} - F$ is an open set.
2. [Baker 2.1.9] Use Definition 2.1.6 to show that $g(x) = \begin{cases} -3 \text{ if } x < 1 \\ 3 \text{ if } x \geq 1 \end{cases}$ is not a continuous function.
3. [Baker 2.1.10] Complete the proof of Theorem 2.1.8.
4. [Baker2.2.7] Show that $\mathscr{U}$ is finer than the finite complement topology for $\mathbb{R}$.
5. [Baker 2.3.13] Let U be a closed set and let V be an open set in a topological space. Show that $U - V$ is closed and $V - U$ is open.
6. [Baker 2.3.15] Let A and B be subsets of a topological space $(X, \mathscr{T})$. Show that

$$X - \mathrm{Cl}(A \cup B) = (X - \mathrm{Cl}(A)) \cap (X - \mathrm{Cl}(B)).$$

7. [Baker 2.4.14] Let A be a subset of a topological space. Prove that $\mathrm{Cl}(A) = \mathrm{Int}(a) \cup \mathrm{Bd}(A)$.
8. [Baker 2.5.10] Let $(X, \mathscr{T})$ be a topological space, $\mathscr{B}$ a base for $\mathscr{T}$, and $A \subseteq X$. Show that $x \in \mathrm{Cl}(A)$ iff for each $B \in \mathscr{B}$ with $x \in B$, $B \cap A \neq \varnothing$.
9. In a space $(X, \mathscr{T})$ any collection of open sets whose union equals X and that is closed under finite intersection is a base for $\mathscr{T}$.