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Question 3

I fear the training of applied mathematicians. It is the engineer and the scientist whose role it is to define the parameters of a physical situation, and then empirically describe it under a given set of rules. Mathematics is the only art in which anything may be thought of, in any way, indiscriminately. A yellow apple may be treated as purple orange. Why not? Vitally, it is the job of the mathematician to fix the underlying cause of every industrial frustration: a stagnant paradigm.

I wonder how many mathematicians think about it this way?

Newton had to invent Calculus (I know a guy in Germany did it too, but for the sake of example...) before he could then use it to describe his model of Newtonian physics, which has subsequently been used in every field of engineering and science. If he had tried to come to form his physical laws without first pursuing pure mathematics, he would have been lost. Lucky for us he was as brilliant a physicist as he was a mathematician, and alone could see the relation between the mathematics and the science.

Still, as the article stated, industry has opened up to applied mathematics (though it borders on what I'd call quasi-engineering). The question is, "Does there need to be a systematic training of these applied mathematicians?". I agree with the author that it is important to be able to communicate both orally and with the pen. Outside of that, as I have alluded to earlier, the mindset of the mathematician in industry is most valuable when free of any constraint. Thus, training in mathematics, which is constantly filled with solving problems under a set of constraints, stunts the potential of the mathematician to develop truly revolutionary means looking at the world.

Just as there is a need for the applied force of thought, so there is a need for the purely abstract. Properly, that is the need that mathematics fills.

Awesome. One could quibble a bit with your historical example, but as you note it serves very well to illustrate your idea. You've packed a heck of a lot into a page - great job.

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In response to the second question, the five requirements suggested here are quite useful when trying to improve the curriculum of a Mathematics department; however, we feel that the Calculus class can be an introduction to these tactics, but not a course in these tactics. Each requirement should be introduced in the following way to a Calculus course.

First, focusing on problem-solving should be pushed in any course, since that is what learning is for...to solve problems. This should be used in a Calculus class in the form of homework, group problems, and much to students disliking, tests. Second, gaining experience in communications should be found in group projects, where the student must talk over the project and agree on an answer. When agreed upon the students must present a well written presentation to the teacher. The two next topics deal with exposure to cognate disciplines and at least one paradigm of applied mathematics research. Both perhaps should be discussed in lecture to a Calculus class, but these are things that should be done mostly after the Calculus class. These are important to the overall outcome of the students, but Calculus should be understood and used to understand these other topics. Lastly is the topic of gaining confidence acquired in open-ended problem solving. This should not be introduced to a Calculus class, this is less about the understanding of Calculus and more to putting the skills to work possibly without the proper understanding of the skills needed to solve such a problem. We believe this is too advanced for practice in a Calculus class.

In general, these ideas are good and though out, but they are meant more for an advanced mathematics student, than the students taking Calculus. These requirements promote creativity, confidence, and presentation, which are all good qualities. We believe Calculus is a basic math course, and therefore could be more effectively used as a tool to build on. If taught and understood well, Calculus can be very useful in more advanced applied studies in areas of not only Math, but also Physics, Engineering, Meteorology, etc. Not to shy away from the introduction of these requirements, Calculus classes should focus on the basics and understanding of the tactics and theories, so that they can be put to use later on.

The benefits of these requirements are that the students would be more appealing to the work force, and would also be more prepared to use applied mathematics. The cost would not be a money factor, due to the courses already being taught, but more of a sanity factor. Taking up time with some of these requirements would lessen the time the student had to focus on the theories and structures of Calculus. These are important and must be understood in order to be used in higher mathematics and other related fields. Since Calculus is a basic course, and should be, it should be used, not without communication and interaction with others, but along with these to teach students the ground work for further knowledge to build on.

This is an absolutely great answer. It's extremely well-reasoned, clearly articulated, and thorough.

Well done.

(2) $T(x,t) = T_0 + T_1 e^{-\lambda x} \sin(wt - \lambda x)$

where $w = \frac{2\pi}{365}$ and λ is a positive constant.

(a) $$\frac{\partial T}{\partial x} = T_1\left[-\lambda e^{-\lambda x}\sin(wt-\lambda x) + e^{-\lambda x}(-\lambda)\cos(wt-\lambda x)\right]$$

$$= -\lambda T_1 e^{-\lambda x}\left[\sin(wt-\lambda x) + \cos(wt-\lambda x)\right]$$

This represents the rate of change of temperature with respect to depth.

(b) $$\boxed{\frac{\partial T}{\partial t} = T_1 e^{-\lambda x} w\cos(wt-\lambda x)}$$

This represents the rate of change of temperature with respect to time.

(c) $$T_{xx} = \frac{\partial^2 T}{\partial x^2} = -\lambda T_1\left[-\lambda e^{-\lambda x}\sin(wt-\lambda x) - \lambda e^{-\lambda x}\cos(wt-\lambda x)\right] +$$

$$-\lambda T_1\left[-\lambda e^{-\lambda x}\cos(wt-\lambda x) + \lambda e^{-\lambda x}\sin(wt-\lambda x)\right]$$

$$T_{xx} = \lambda^2 T_1 e^{-\lambda x}\left[\sin(wt-\lambda x) + \cos(wt-\lambda x) + \cos(wt-\lambda x) - \sin(wt-\lambda x)\right]$$

$$= 2T_1\lambda^2 e^{-\lambda x}\cos(wt-\lambda x)$$

$$\frac{T_t}{T_{xx}} = \frac{T_1 e^{-\lambda x} w\cos(wt-\lambda x)}{2T_1\lambda^2 e^{-\lambda x}\cos(wt-\lambda x)} = \frac{w}{2\lambda^2}.$$

$$\boxed{T_t = \left(\frac{w}{2\lambda^2}\right) T_{xx}}$$

great

where $w = \frac{2\pi}{365}$ and λ is a positive constant

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In[86]:= Plot3D[10 E^-0.2x Sin[1/365 (2 π) y - 0.2 x], {x, 0, 40}, {y, 0, 2*365}, PlotPoints → 50,
     PlotRange → {-10, 10}, BoxRatios → {1, 1, 1}, AxesLabel -> {"x", "t", "T"},
     ViewPoint -> {1.5, -3, 1}]
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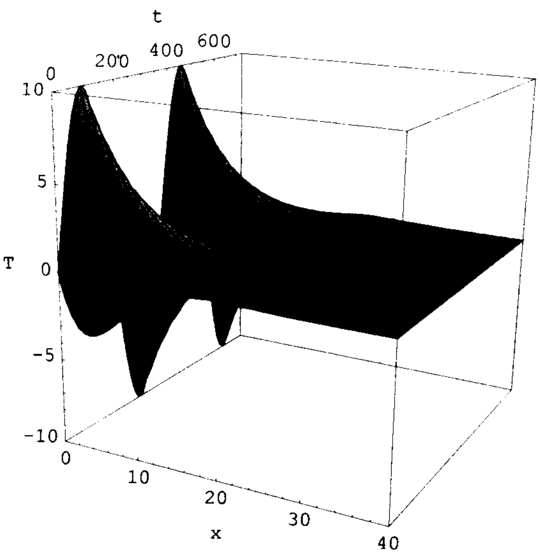
(a)



Nice.

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Out[86]= - SurfaceGraphics -
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(b) In the expression sin(ωt − αx), the term −αx is a phase shift that depends upon depth. It signifies the physical reality that temperature fluctuation lags at depth. For instance, the earth ten feet underground is insulated by all the overlying material; relative to the soil at, say, one-foot depth, the deeper earth will take a long time to respond to a change in surface temperature. The greater the depth, the slower the response: hence the negative sign.

Excellent

3. by definition: $f_x := \lim_{h \to 0} \dfrac{f(x+h,y) - (f(x,y))}{h}$

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$$f_x(0,0) = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$f_x(0,0) = \lim_{h \to 0} \frac{\sqrt[3]{(0+h)^3 + 0^3} - \sqrt[3]{0^3 + 0^3}}{h}$$

$$f_x(0,0) = \lim_{h \to 0} \frac{\sqrt[3]{h^3}}{h} = 1$$

Nice

4. $Q := \dfrac{(\pi \cdot p \cdot r^4)}{8 \cdot l}$ $\quad Q_r := \dfrac{(\pi \cdot p \cdot r^3)}{2 \cdot l}$ $\quad Q_p := \dfrac{(\pi \cdot r^4)}{8 \cdot l}$

$$\frac{dQ}{dt} := \left[\frac{(\pi \cdot p \cdot r^3)}{2 \cdot l}\right] \cdot .04 + \left[\frac{(\pi \cdot r^4)}{8 \cdot l}\right] \cdot .09$$

Good

and assuming p=0 for differential pressure $\dfrac{dQ}{dt} := 17.25 \cdot 10^{-6} \quad \dfrac{atm \cdot cm^3}{s}$

5. Vs = LWH + 2(WH + LW + LH) + π(L + W + H + 4/3) as shown by below diagram.

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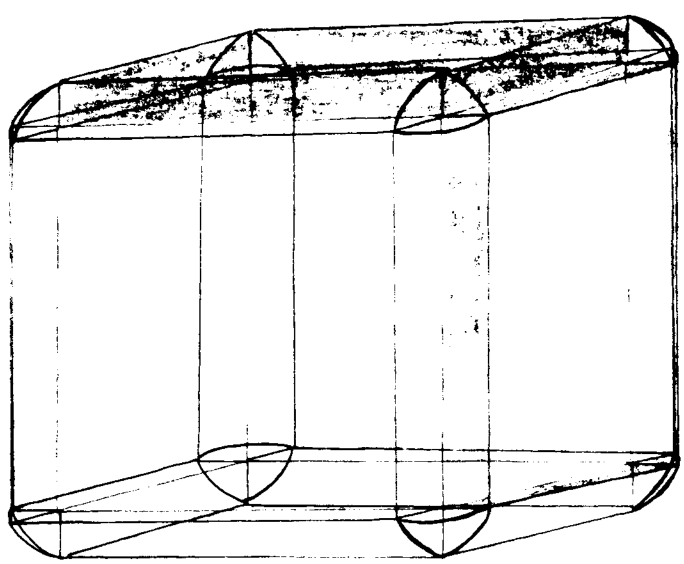
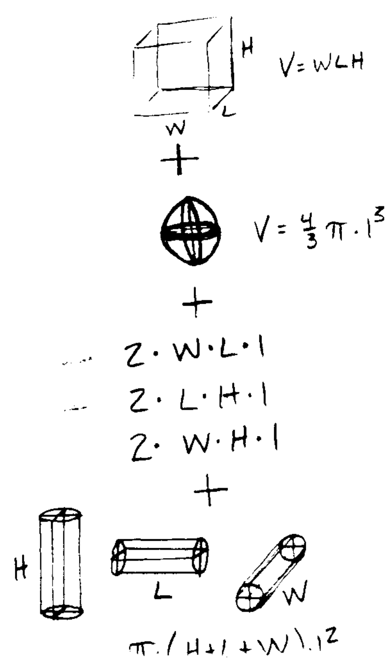
Great





#4. Q: Find $\frac{dQ}{dt}$

Given: $Q = \frac{\pi p r^4}{8\ell}$

$r = 0.11$ cm

$\ell = 0.3$ cm

$p = 0$

$\frac{dr}{dt} = 0.04$ cm/sec

$\frac{dP}{dt} = 0.09$ atmospheres/sec

Use the helpful tree! ☺

$$\begin{tikzcd} & Q \arrow[dl, "\frac{\partial Q}{\partial r}"'] \arrow[dr, "\frac{\partial Q}{\partial P}"] & \\ r \arrow[d, "\frac{dr}{dt}"'] & & P \arrow[d, "\frac{dP}{dt}"] \\ t & & t \end{tikzcd}$$

Great

$$\frac{\partial Q}{\partial r} = \frac{4\pi p r^3}{8\ell}$$

$$\left(\frac{\partial Q}{\partial r}\right)\cdot\left(0.04\right) = \frac{4(\pi)(0)(r^3)}{8\ell} = \underline{0}$$

$$\frac{dr}{dt} = 0.04 \qquad \frac{\partial Q}{\partial r}\cdot\frac{dr}{dt} = (0)\cdot(0.04) = 0$$

$$\frac{\partial Q}{\partial P} = \frac{\pi r^4}{8\ell} = \frac{(\pi)(0.11)^4}{[8(0.3)]} = 1.917 \times 10^{-4}$$

$$\frac{dP}{dt} = 0.09 \qquad \frac{\partial Q}{\partial P}\cdot\frac{dP}{dt} = (1.917 \times 10^{-4})\cdot(0.09)$$

$$= \underline{1.725 \times 10^{-5}}$$

$$\therefore \frac{dQ}{dt} = \underline{1.725 \times 10^{-5}}$$

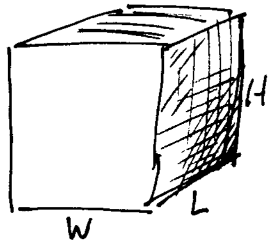
(Poor li'l vole! ☹)

ublem Set 1. Problem #5

Problem: Let B be a solid box with length L, width W, and height H. Let S be the set of all points that are a distance of at most 1 from some point of B. Express the volume of S in terms of L, W, + H.

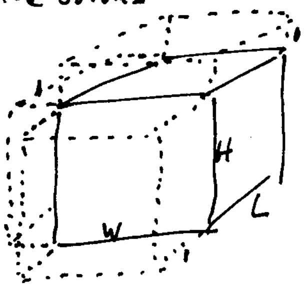
Solution:

First, we have the volume of B:



The volume of B is $\underline{LWH}$.

Second, we have the volume of the solid created by extending each face of B by 1:



This obviously creates two blocks of volume $LW(1)$, two of volume $LH(1)$, and two of volume $WH(1)$.
This gives a total volume for the six blocks of $\underline{2LW + 2LH + 2WH}$

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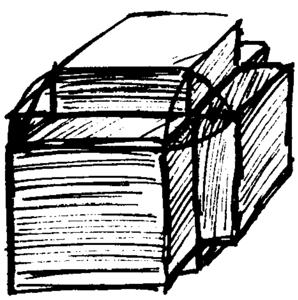
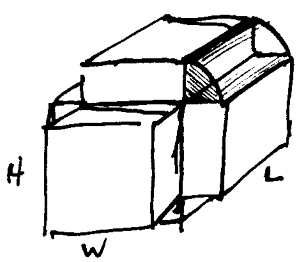
Excellent

Next, we determine the volume formed by the partial spheres at the corners of B.



Obviously, there are 8 $\frac{1}{8}$ spheres on the outside of B. Since there are 8, the total volume of the ~~spheres is~~ corners of B is $\underline{\frac{4}{3}\pi}$

Lastly, Determine the rounded edges formed between the extensions of the faces of the box.



For each axis ($L, W, + H$) there will be 4, $\frac{1}{4}$ cylinders of the length of the axis. Since there are 4, the volume will be the volume of 3 cylinders with heights $L, W, + H$. Thus volume is: $\pi H + \pi W + \pi L$

$\therefore$ The total volume is: $LWH + 2LW + 2LH + 2WH + \frac{4}{3}\pi + \pi H + \pi L + \pi W$