Calculus IV Quiz 2 Spring 1999 2/12/99

1. Show that $\int_0^1 x e^{x+y}\,dx = e^y$. [Hint: Integrate by parts.]

$u = x \qquad dv = e^{x+y}dx$

$du = dx \qquad v = e^{x+y}$

$\int e^{x+y}dx = \int e^p dp = e^p + C = e^{x+y} + C$

$p = x+y$

$\frac{dp}{dx} = 1$

$dp = dx$

$x e^{x+y}\Big|_0^1 - \int_0^1 e^{x+y}dx = e\,e^y - e^{x+y}\Big|_0^1 = ee^y - ee^y + e^y =$

$x e^{x+y}\Big|_0^1 = 1e^{1+y} - 0 = e^1 \cdot e^y$

$e^{x+y}\Big|_0^1 = e^{1+y} - e^y$

$\boxed{e^y}$

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1. Show that $\int_0^1 xe^{x+y}dx = e^y$. [Hint: Integrate by parts.]

$$\int_0^1 xe^x e^y dx = e^y \int_0^1 xe^x dx = e^y \left[x \cdot e^x - \int_0^1 e^x dx \right]$$

Nice Move!

$u = x \quad dv = e^x dx$

$du = dx \quad v = e^x$

$$= e^y \left[xe^x - e^x \right]_0^1$$

$$= e^y \left[[e - e] - [0 - e^0] \right]$$

Beautiful!

$$= e^y [0 + e^0]$$

$$= e^y [0 + 1] = 0 + e^y =$$

$$= e^y$$

2. Show that $\int_0^{\sqrt{\pi}} x\sin(x^2+y^2)\,dx = \frac{-1}{2}\left[\cos(\pi+y^2)-\cos(y^2)\right]$. [Hint: Try a u substitution.]

let $u = x^2+y^2$

$\frac{du}{dx} = 2x$

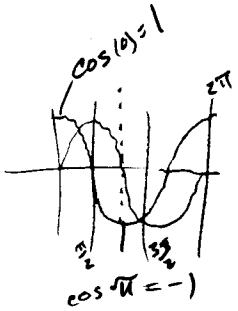
$\frac{du}{2x} = dx$

$x=\sqrt{\pi} \rightarrow u = \pi+y^2$

$x=0 \rightarrow u = y^2$

} change limits of integration

$$\int_{y^2}^{\pi+y^2} \frac{x\sin u\,du}{2x} = -\frac{1}{2}\cos u \Big|_{y^2}^{\pi+y^2}$$



$$= \left[-\frac{1}{2}\left(\cos(\pi+y^2)-\cos(y^2)\right)\right]$$

180° phase shift (π shift)

$\rightarrow \cos(\pi+y^2) = -\cos(y^2)$

$$-\frac{1}{2}\left(-\cos y^2 - \cos y^2\right)$$

$$= -\frac{1}{2}(-2\cos y^2)$$

$$= \left[\cos(y^2)\right]$$

Bonus!