$\frac{10}{10}$ Key

Calculus IV Quiz 4 Spring 1999 4/5/99

1. Compute $\int_C (x^2 y i - xy j)\cdot dr$ for the path C given by $r(t) = t^3 i + t^4 j$ for values of t between 0 and 1.

$\int_C F(r(t))\cdot r'(t)\,dt$

$F = \langle x^2 y, -xy\rangle$

$r(t) = \langle t^3, t^4\rangle$

$r'(t) = \langle 3t^2, 4t^3\rangle$

$F(r(t)) = F(t^3, t^4)$
$= \langle t^6 \times t^4, -t^3 \times t^4\rangle$
$= \langle t^{10}, -t^7\rangle$

$\int_0^1 \langle t^6 t^4, -t^3 t^4\rangle \cdot \langle 3t^2, 4t^3\rangle\,dt$
$= \int_0^1 \langle t^{10}, -t^7\rangle\cdot\langle 3t^2, 4t^3\rangle\,dt$
$= \int_0^1 (3t^{12} + (-4t^{10}))\,dt$
$= \frac{3}{13}t^{13} + (-\frac{4}{11}t^{11})\Big|_0^1 = \frac{3}{13} - \frac{4}{11}$
$= \frac{33}{143} - \frac{52}{143} = \frac{-19}{143}$

2. Compute $\int_C (2xy^3 i + 3x^2 y^2 j)\cdot dr$ where C is the bottom arc of a circle (centered at the origin) of radius 7 from the point (0,7) to the point $\left(\sqrt{\frac{7}{2}}, \sqrt{\frac{7}{2}}\right)$ traversed counterclockwise.

Potential function
$F(x,y) = x^2 y^3$

$\nabla f = \langle 2xy^3, 3x^2y^2\rangle$

$(0,7)$ to $\left(\sqrt{\frac{7}{2}}, \sqrt{\frac{7}{2}}\right)$

$F(\vec{r}(b)) - F(\vec{r}(a)) = F\left(\sqrt{\frac{7}{2}}, \sqrt{\frac{7}{2}}\right) - F(0,7)$
$= \frac{7}{2}\cdot\left(\sqrt{\frac{7}{2}}\right)^3 - 0$
$= \frac{49}{4}\cdot\sqrt{\frac{7}{2}}$