Calculus IV Quiz 6 Summer 2000 7/17/2000

1. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x,y)=(2x-y)\mathbf{i} + (3y^2-x)\mathbf{j}$ where C is the first-quadrant portion of a circle of radius 3 traversed counterclockwise.

5

Assume $\vec{F} = \nabla f$ and $f_x = 2x-y$ $f_y = 3y^2 - x$ since $f_{xy} = f_{yx} = 1$ $\vec{F}$ is a conservative vector field.

$f = x^2 - yx$ or $y^3 - yx \Rightarrow x^2 - yx + y^3$

$(x_f, y_f) = (0,3)$ $(x_i, y_i) = (3,0)$

so from Fun. Thrm of Line Integrals

$$\int_C \vec{F}\cdot d\vec{r} = f(x_f,y_f) - f(x_i,y_i)$$

$$= [0^2 - 3\cdot 0 + 3^3] - [3^2 - 3\cdot 0 + 0^3] = 27 - 9 = 18$$

2. Compute $\int_C \langle 2x+y, 3x^2\rangle \cdot d\mathbf{r}$ for C the line segment beginning at (0,1) and ending at (2,4).

5

$x(t) = 0 + 2t$

I $y(t) = 1 + 3t$ $\quad 0 \le t \le 1$

$r(t) = \langle 2t, 1+3t \rangle$

III $r'(t) = \langle 2, 3 \rangle$

$\frac{\partial x}{\partial y} = 1 \neq \frac{\partial y}{\partial y} = 6x$

II $\vec{F}(\vec{r}(t)) = \langle 2(2t) + (1+3t), 3(2t)^2 \rangle$

$= \langle 4t + 1 + 3t, 12t^2 \rangle$

$= \langle 7t+1, 12t^2 \rangle$

IV $\int_0^1 \langle 7t+1, 12t^2 \rangle \cdot \langle 2, 3 \rangle \, dt = \int_0^1 (14t + 2 + 36t^2) \, dt$

$= 7t^2 + 2t + 12t^3 \Big|_0^1 = 7 + 2 + 12 = \boxed{21}$

Great