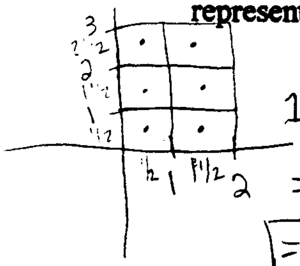
Exam 2 Calc IV (Math2443-002) 7/3/2002

Each problem is worth 10 points. Show all work for full credit. Please circle all answers and keep your work as legible as possible.

1. Estimate the volume of the solid that lies below the surface $z=x^2+4y$ and above the rectangle $R=\{(x,y)|0\le x\le 2, 0\le y\le 3\}$. Use a Riemann sum with m=2, n=3, and use the midpoint as the representative for each subrectangle.



$\Delta x \Delta y = 1$

$1\left[f(\tfrac{1}{2},\tfrac{1}{2}) + f(\tfrac{1}{2},\tfrac{3}{2}) + f(\tfrac{1}{2},\tfrac{5}{2}) + f(\tfrac{3}{2},\tfrac{1}{2}) + f(\tfrac{3}{2},\tfrac{3}{2}) + f(\tfrac{3}{2},\tfrac{5}{2})\right]$

$= (\tfrac{1}{2})^2 + 4(\tfrac{1}{2}) + (\tfrac{1}{2})^2 + 4(\tfrac{3}{2}) + (\tfrac{1}{2})^2 + 4(\tfrac{5}{2}) + (\tfrac{3}{2})^2 + 4(\tfrac{1}{2}) + (\tfrac{3}{2})^2 + 4(\tfrac{3}{2}) + (\tfrac{3}{2})^2 + 4(\tfrac{5}{2})$

$\boxed{= \frac{87}{2}}$

Good

10

2. Suppose that projections for this year's wheat harvest in the state of Kansas are 4000 bushels per square mile at the southwest corner, 8000 bushels per square mile at the southeast corner, and 9000 bushels per square mile at the northeast corner. You may also assume that the wheat production varies linearly, and that Kansas is close enough to being a rectangle 200 miles from south to north and 400 miles from west to east. Set up a double integral to compute the total wheat production of Kansas.

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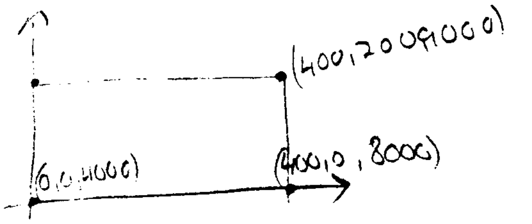


$m = \frac{8000-4000}{400-00} = 10$

$n = \frac{9000-8000}{200-0} = 5$

$z - z_0 = m(x-x_0) + n(y-y_0)$ Great

$z - 4000 = 10x + 5y \Rightarrow z = 10x + 5y + 4000$

$$\boxed{\int_{x=0}^{x=400}\int_{y=0}^{y=200} 10x + 5y + 4000\, dy\, dx}$$

3. Pat the mathematician is proposing a new public wading pool be built one foot deep in the shape of the curve given in polar coordinates by the equation $r = 10 + 2\sin(5\theta) + \cos(7\theta)$. Pat claims that a pool with such a mathematically perfect shape will enhance childrens' understandings of spatial relationships and eventually make them better at calculus. In order to apply for a government grant to fund the pool, Pat needs to compute the volume of the pool. **Set up** an iterated integral for the volume of water contained in such a pool.

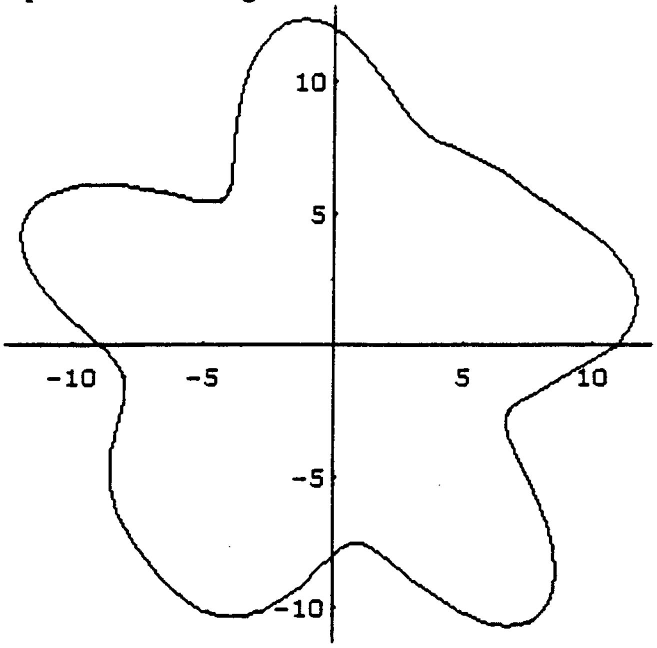
with a triple integral, use 1 to get volume

$$\iiint 1 \, r\,dr\,d\theta\,dz$$

Yes

$$\boxed{\int_{z=0}^{z=1}\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=10+2\sin 5\theta+\cos 7\theta} r\,dr\,d\theta\,dz}$$

W



The graph of $r=10+2\sin 5\theta+\cos 7\theta$ for $0\le\theta\le 2\pi$

4. Evaluate $\iiint_E z\,dV$, where E lies between the spheres $x^2+y^2+z^2=1$ and $x^2+y^2+z^2=4$ in the first octant.

Spherical coord. Yes!

W

$$\sqrt{x^2+y^2+z^2}=\sqrt{4}=2$$

$$\sqrt{x^2+y^2+z^2}=\sqrt{1}=1$$

$$\iiint_E z\,dv$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho\cos\phi\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_1^2 \rho^3\cos\phi\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\int_1^2 \rho^3\,d\rho \quad \int_0^{\pi/2}\cos\phi\sin\phi\,d\phi \quad \int_0^{\pi/2} d\theta$$

Excellent!

$$= \left[\frac{\rho^4}{4}\right]_1^2 \cdot \left[\frac{1}{2}\sin^2\phi\right]_0^{\pi/2} \cdot [\theta]_0^{\pi/2}$$

$$= \left[4-\frac{1}{4}\right] \cdot \left[\left(\frac{1}{2}\cdot 1\right)-0\right] \quad \left[\frac{\pi}{2}\right]$$

$$= \left[\frac{15}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}\right] = \boxed{\frac{15\pi}{16}}$$

5. **Set up** integrals which would give the center of mass of a flat sheet (with constant density) shaped like the region bounded by the curves $y=x^3$, $x=1$, and $y=0$.

density $= k$

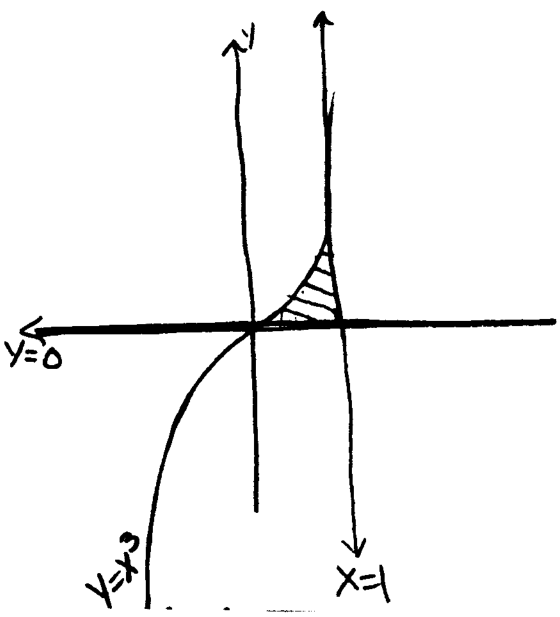
$$m = \int_0^1 \int_0^{x^3} k \, dy \, dx$$

$$M_x = \int_0^1 \int_0^{x^3} k \cdot y \, dy \, dx$$

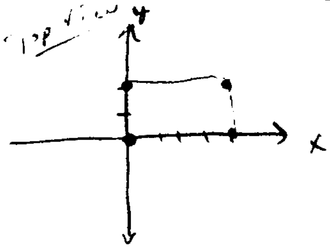
$$M_y = \int_0^1 \int_0^{x^3} k \cdot x \, dy \, dx$$

$$\bar{x} = \frac{M_y}{m} \qquad \bar{y} = \frac{M_x}{m}$$

Great



6. **Set up** an iterated integral for the **surface area** of the part of the cylinder $y^2 + z^2 = 9$ that lies above the rectangle with vertices (0,0), (4,0), (0,2), and (4,2).



$$z = \sqrt{9-y^2}$$

$$f_x = \frac{\partial z}{\partial x} = 0$$

$$f_y = \frac{\partial z}{\partial y} = \tfrac{1}{2}(9-y^2)^{-1/2}(-2y) = -\frac{y}{\sqrt{9-y^2}}$$

$$A(s) = \int_0^4 \int_0^2 \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \, dy \, dx$$

$$A(s) = \int_0^4 \int_0^2 \sqrt{1 + (0)^2 + \left(-\frac{y}{\sqrt{9-y^2}}\right)^2} \, dy \, dx$$

Well done

7. Jeb is a Calc 4 student at O.S.U., and he's getting frustrated with double integrals. Jeb says "Heck, I can't figure none of this order stuff out. The teacher says sometimes ya gotta do it dx then dy, and sometimes ya gotta do it dy then dx. I figure if it works one time, it oughta always be the same. The teacher just keeps sayin' ya gotta look at the graph and all, but I'm not much one for graphs. I'm fixin' to just do the whole test with the dx first and the dy second, 'cause I figure that's always gotta work."

Explain to Jeb, in terms he can understand, whether his plan will always work, or in which specific instances it might not.

Well Jeb, the difference in using dy dx or dx dy is as simple as this. One might take 2 minutes and the other might take 2 hours, 4 aspirin, a large computer, & 7 proffessors. :) There can often be a big difference. Take the integrall

$\iint_R \sqrt{4y^2+2}\, dA$, which order should we go Jeb, dx ... or dy dx?

Say you go dy dx, tell me how to integrate that with respect to y! No offence, but I'm stupid and I can't do it :), Now do it dx dy OK! $\int_0^1 \left[x\sqrt{4y^2+2} \Big|_0^y \right] dy$ - That's much more simple, you're left with all y's and you can do a simple u substitution. There are times, where it just might not matter, but always look at the graph. Some times left to right is much simpler than down to up or vica versa. Listen to your smart teacher Jeb!

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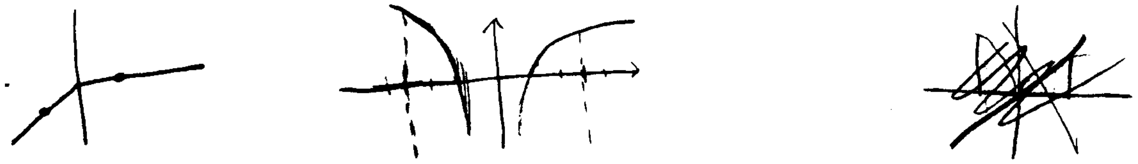
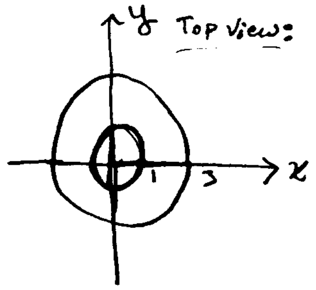
Excellent



8. **Set up** an iterated integral for the volume of the solid bounded by the surface $z=\ln(x^2+y^2)$, the cylinder $x^2+y^2=9$, and the plane $z=0$.

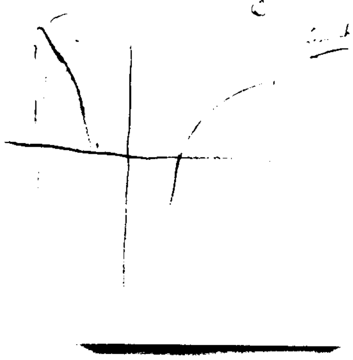
Top View:



Excellent

$$\int_0^{2\pi}\int_1^3\int_0^{\ln r^2} r\,dz\,dr\,d\theta$$

$\ln(x^2+y^2)=3$



9. Use the transformation $x=2u+3v$, $y=3u-2v$ to evaluate the integral $\iint_R (x+y)\,dA$, where R is the square with vertices (0,0), (2,3), (5,1), and (3,-2) [Note: if the Jacobian is negative, just strip off the minus sign].

W

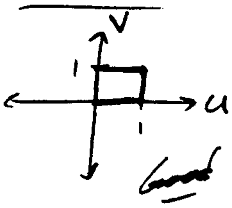
$x = 2u+3v$
$y = 3u-2v$

$$\text{Jacobian} = \begin{vmatrix} \partial x/\partial u & \partial y/\partial u \\ \partial x/\partial v & \partial y/\partial v \end{vmatrix} = \begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = -4-9 = -13$$

$\Rightarrow 13$

$x+y = (2u+3v)+(3u-2v)$
$= 5u+v$

$(x,y)=(0,0)$	$(x,y)=(2,3)$	$(x,y)=(5,1)$	$(x,y)=(3,-2)$
$0=2u+3v$	$2=2u+3v$	$5=2u+3v$	$3=2u+3v$
$0=3u-2v$	$3=3u-2v$	$1=3u-2v$	$-2=3u-2v$
$(u,v)=(0,0)$	$(u,v)=(1,0)$	$(u,v)=(1,1)$	$(u,v)=(0,1)$



Good

$$\Rightarrow \iint_R (x+y)\,dA = \int_0^1\int_0^1 5u+v\,(13)\,du\,dv$$
$$= 13\int_0^1 \left[\tfrac{5}{2}u^2+uv\right]_0^1 dv$$
$$= 13\int_0^1 \left(\tfrac{5}{2}+v\right)dv$$
$$= 13\left[\tfrac{5}{2}v+\tfrac{1}{2}v^2\right]_0^1$$
$$= 13\left(\tfrac{5}{2}+\tfrac{1}{2}\right)$$
$$= 13(3)$$
$$= \boxed{39}$$

Well done