Problem Set #1

1. Find an equation of the plane tangent to f(x,y)= $x^2 + y^2$ at the point (1,2). Have Mathematica produce a graph of f along with the tangent plane, and make sure you get a nice view of the point of tangency.

SOLUTION:

To begin this problem, let's first take a look at the graph of the function

$$f(x,y) = x^2 + y^2$$

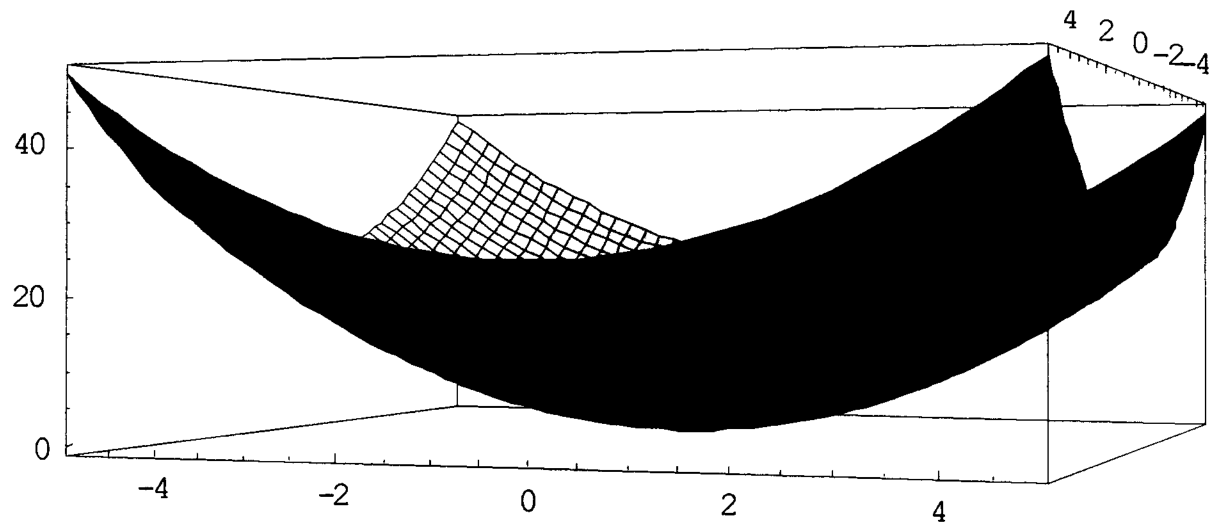

Figure 1a. Produced by Mathematica from a viewpoint oj (3,0,1).

To begin our search for one of the infinite tangent planes to this paraboloid, we need to consider the partial derivatives of the function with respect to both x and y. Once we have our partials, we can find the slope of the tangent lines at our given points and use the equation for tangent planes in three space,

$$z - z_0 = m(x - x_0) + n(y - y_0)$$

where m and n are the slopes of the respected partials, and x_0, y_0, and z_0 is our point of tangency.

First, we find the partials.

$$f(x,y) = x^2 + y^2$$

$$\frac{\partial}{\partial x}(x^2+ y^2) = 2x$$

$$\frac{\partial}{\partial y}(x^2 + y^2) = 2y$$

Taking our partial derivative results, we now substitute in the point (1,2) to our partials get x_0 and y_0. To find z_0 on our tangent plane, we'll plug m and n back into our original equation.

$$m = f_x\,(1,2) = 2(1)= 2.$$

$$n = f_y\,(1,2) = 2(2) = 4.$$

$$z_0 = (m^2 + n^2) = (1^2 + 2^2) = 5.$$

We know have all the required variables for our tangent plane, so we simply plug in,

$$z - z_0 = m(x-x_0) + n(y-y_0)$$

$$z - 5 = 2(x-1) + 4(y-2),$$

Excellent

and that is our tangent plane. Now, let's look at the plane in three space.

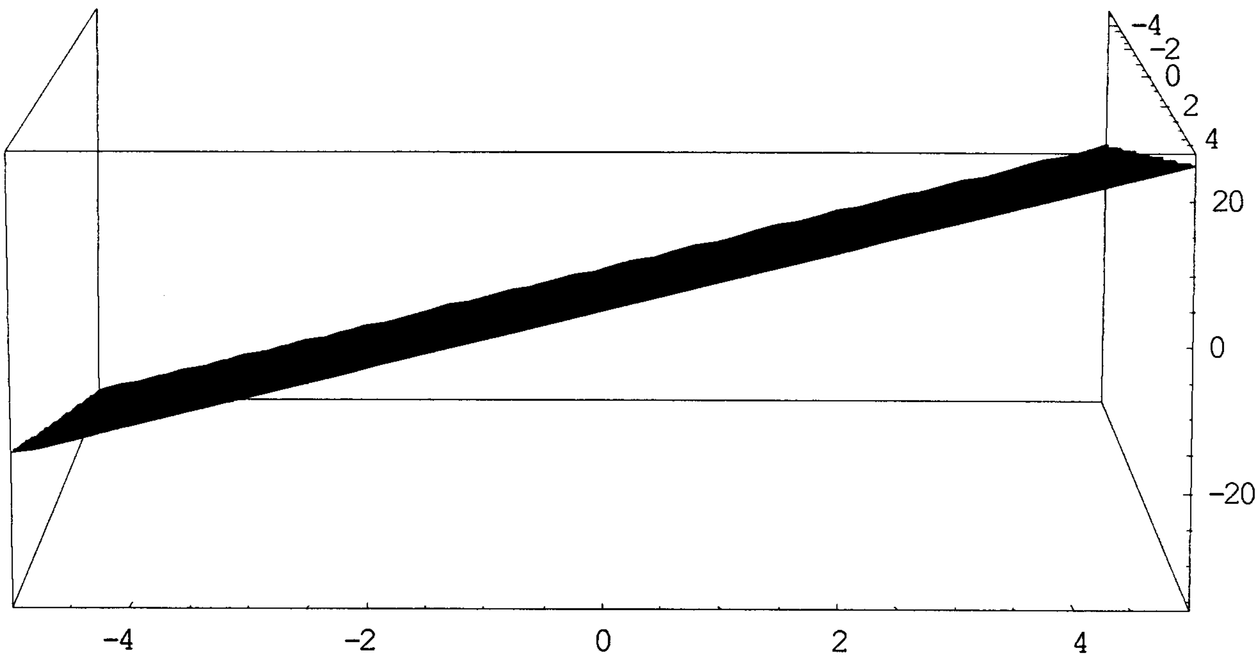

Figure 1b. Our tangent plane to the graph of $f(x,y) = x^2 + y^2$ *from a viewpoint of* (6,0,1).

2. Describe (as if you were trying to convey it to someone over a telephone) the graph of $f(x,y) = \ln(x^2 + y^2)$. Be sure to include an accurate description of the graph's behavior near the origin.

It is a funnel shape plate. The neck part of the funnel is the bottom of the plate. As it gets closer to the origin, it starts forming a circle. As long as $x^2 + y^2$ is positive real number, then z will always exist. Yes

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Plot3D[Log[(x^2) + (y^2)], {x, -.5, .5}, {y, -.5, .5},
 PlotPoints → 100, Mesh → False]
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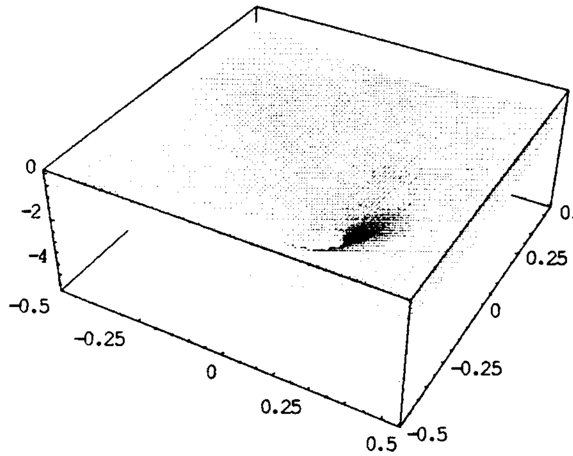

Nice

```
Plot3D[Log[(x^2) + (y^2)], {x, -.5, .5}, {y, -.5, .5},
 PlotPoints → 100, ViewPoint → {0, 0, 2}, Mesh → False]
```

Using Mathematica, we are able to combine the graphs of the function and the tangent plane, showing us clearly the point of tangency.

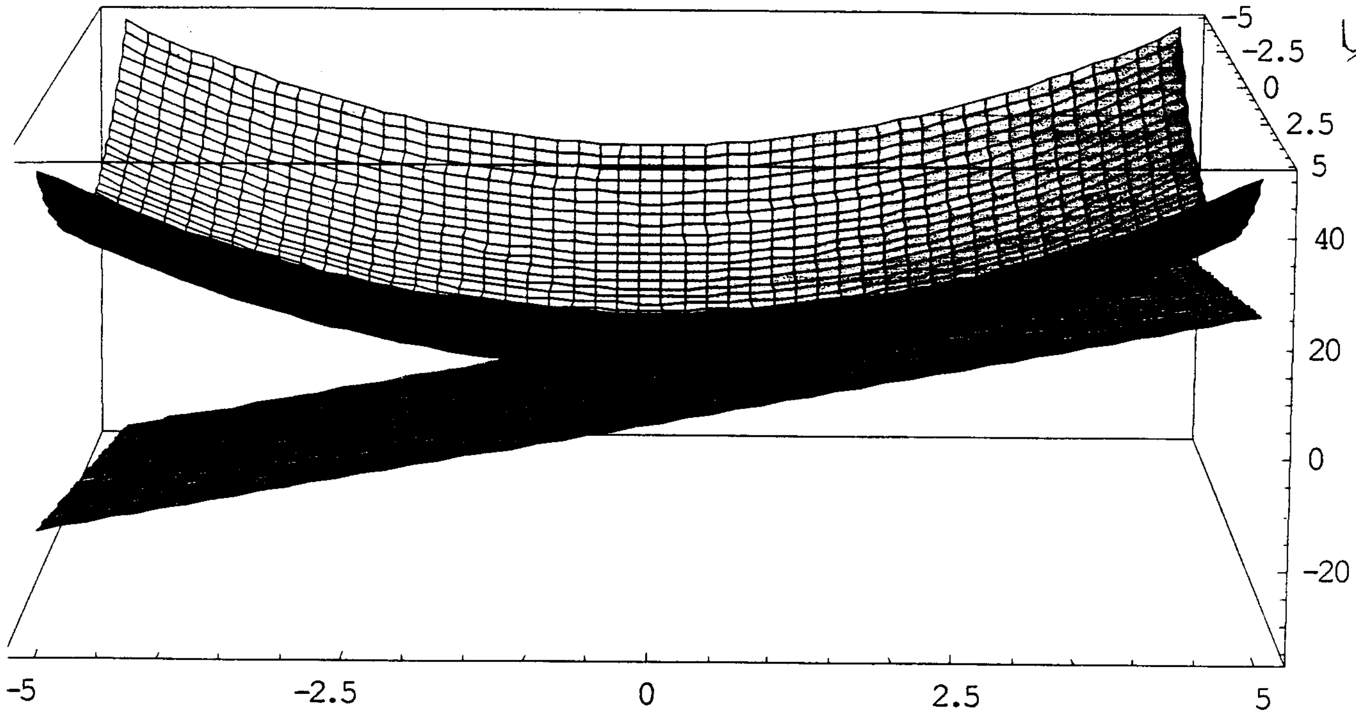

Figure 1c. The graph of $f(x,y) = x^2 + y^2$ *and the tangent plane* $z\text{-}5 = 2(x\text{-}1) + 4(y\text{-}1)$.

4. The work for this problem is on the sheet of notebook paper. For y = x^3, the limit of the equation given is as both x and y approach 0 is 1/2 or .5. Since all the limits given in the problem all are equal to zero but this limit I found is equal to .5, then this means that the actuall limit does not exist. Using the command line

```
Plot3D[(x^3*y)/(x^6+y^2), {x, -5, 5}, {y, -5, 5}, PlotPoints → 80, ViewPoint → {
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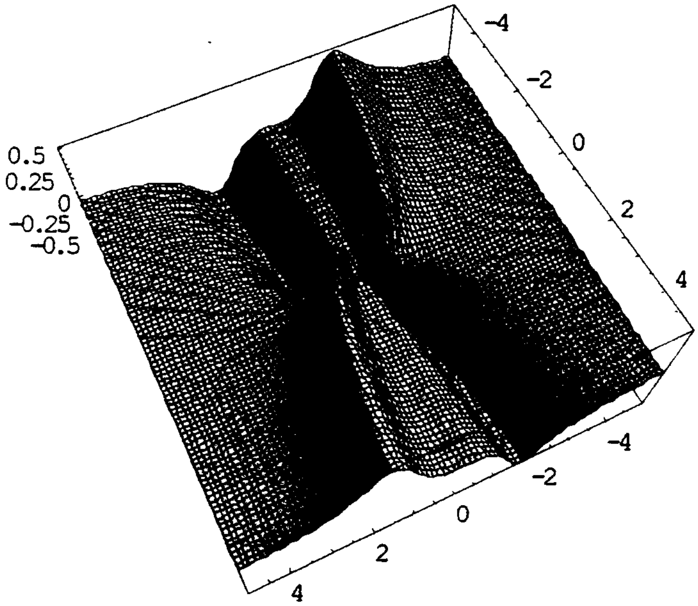

produces the surface which shows that following the trace of y=x^3 is a ridge with a height (or z-value) of .5 which is what we followed and as it approaches y = 0, it maintains its height. Also, one will notice the reverse effect of a trough the follows the trace y = - x^3 which has a height of -.5. As for the rest of the surface, it is level with all lines having a limit of zero as they approach y = 0.

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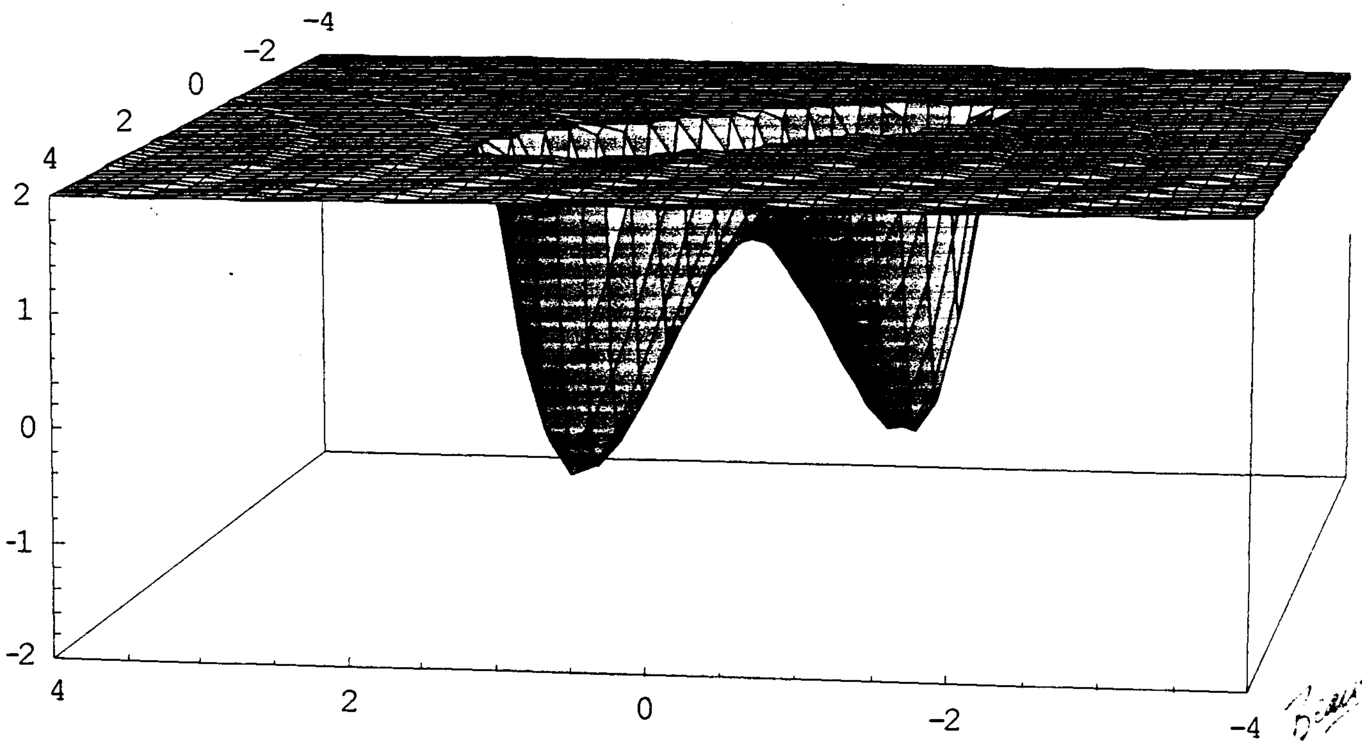
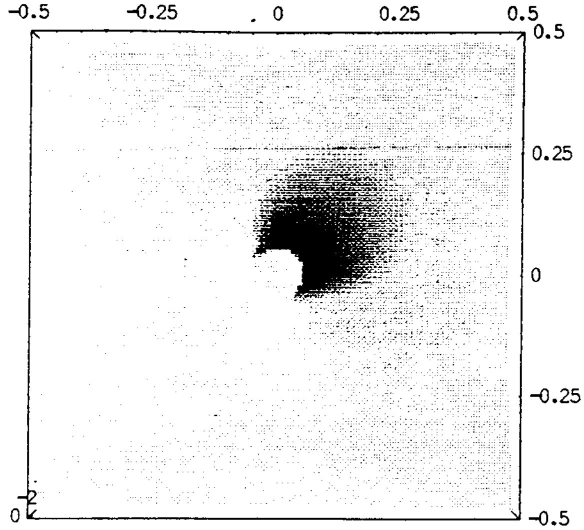
Perfect



5.

Beautiful!

Figure 3: The graph of $f(x,y) = x^4 + y^4 - 4xy + 1$, *clearly showing the local minimums at* $f(1,1)$ *and* $f(-1,-1)$, *and the adjacent saddle point at* $f(0,0)$.




```
Plot3D[Log[(x^2) + (y^2)], {x, -.5, .5}, {y, -.5, .5},
PlotPoints → 50, ViewPoint → {0, 1, 0}, Mesh → False]
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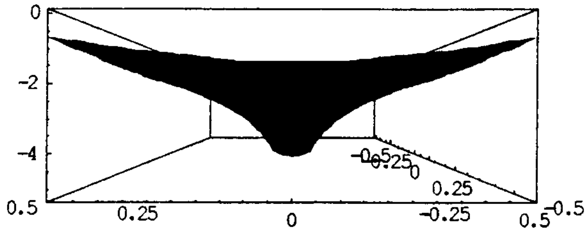
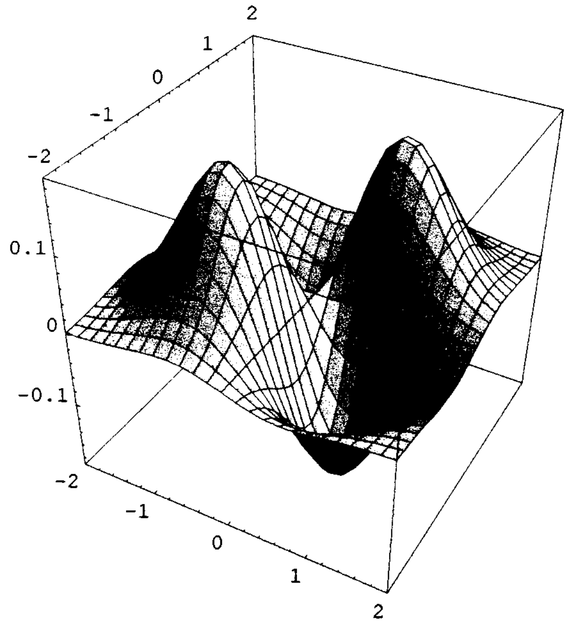


Out[40]= - SurfaceGraphics -

#5

In[13]:= **Plot3D[x*y*E^(-x^2 - y^2), {x, -2, 2}, {y, -2, 2}, PlotPoints → 25, BoxRatios → {1, 1, 1}]**



Out[13]= - SurfaceGraphics -

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5. From the graphs shown, the maximum values occur at (0.7, 0.7, .18) and (−0.7, −0.7, .18) and the minimum values occur at (−0.7, 0.7, −.18) and (0.7, −0.7, .18).

Great